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Fitting discrete probabilistic frequency distributions to disturbed traffic flow at an unsignalized urban intersection

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FITTING DISCRETE PROBABILISTIC FREQUENCY DISTRIBUTIONS TO DISTURBED TRAFFIC FLOW AT AN UNSIGNALIZED URBAN INTERSECTION

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ABSTRACT

Traffic variables, such as volume, speed, gaps and lags, and headways can be described by probability distributions. In this study vehicle arrivals from three directions have been recorded at a four-leg, right-angled, at-grade intersection in Rolla, Missouri, and statistically analyzed. Two distinct probabilistic statistical concepts were used, the continuous (gap) and discrete (counting) distribution. The negative exponential distribution is discussed briefly, whereas the applicability of discrete statistical models for the traffic situation selected is elaborated in detail.

The purpose of this research was to investigate the impairment and distortion that various interferences such as upstream traffic lights, bottlenecks or other STOP sign controlled intersections would have upon approximative discrete frequency functions.

Traffic was counted and recorded on Friday, July 7, 1972, during three two-hour time periods, in the morning, at noon, and during the afternoon rush hours. Fifteen different situations, six each for the North and West approaches, and three for arrivals from the East direction were evaluated. For the counting distribution, vehicular arrivals were grouped arbitrarily into time intervals of 30 and 60 seconds for the North approach leg, because of the slightly higher traffic volumes, 60 and 120 seconds

for the West, and 120 seconds only for vehicles approaching from the East.

The "Goodness of Fit" for acceptable approximation was tested by Pearson's Chi-square for the fifteen situations described before. Then, on the basis of the mean-variance criterion, a choice was made between the binomial and negative binomial distribution. The randomness of the observed frequencies was tested against the characteristics of these distributions again by the Chi-square test and compared with the results of the Poisson fit for acceptableness or rejection of hypotheses at appropriate predetermined significance levels.

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I. INTRODUCTION

A. Statement of Problem

The operational performance of an unsignalized urban intersection depends upon a variety of highly contingent, interrelated complexities, so called"stochastic" phenomena. These include driver behaviour, vehicle types, nearby traffic signals, restrictive physical features, location within a metropolitan area, load factors of adjacent signalized intersections, volume-capacity ratio, and weather and environmental conditions. Variable phenomena of this type in most instances can be analyzed by methods of probability and statistics providing a means for extrapolation.

The adaptability of continuous as well as discrete probability models to various problems in traffic engineering has been shown in numerous cases. The distribution of time headways, gaps and lags* between successive vehicles or delay characteristics are of substantial importance to capacity computations, weaving determinations, merging maneuvers, safety from rear-end collisions, signal timing, in warrants for STOP signs, and in determining whether vehicles waiting at a STOP or YIELD sign can safely enter a traffic stream.

To a large extent vehicular arrivals at an intersection, whether signalized, signed or uncontrolled, are

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* Refer to Table I for terminology

found to be random; that is, any given element of time is as likely to contain an arrival as any other equal element. It is understood, of course, that successive vehicle arrivals constitute an arrival chain being composed of various time spacings (time headways or gaps) between vehicles. In considering vehicle arrivals at an intersection, one has no way of knowing how many vehicles did <u>not</u> arrive. Generally, in traffic situations of this type, the Poisson distribution or one of its more sophisticated modifications should be given consideration for applicability.

Currently, the binomial type statistics are widely recognized for their close and satisfactory approximation to actual field observations in various traffic flow problems. Statistical models most suitable for representing observations on a continuous scale are the normal distribution, the log-normal distribution and the negative exponential distribution. Whereas, if the scale implies discontinuity, the basic principle is a breakdown of data into discrete classes, and a model which embraces this aspect is more appropriate. Typical examples are the binomial, Poisson, negative binomial, and geometric distribution. These will be considered as suitable alternatives for counts where observations are subdivided into mutually exclusive classes.

The application of the Poisson statistical model seems to be very well justified in situations of low free-flowing traffic volumes. However, the usefulness of its application in urban areas with various interferences, disturbances and impairments to traffic movement is questionable. One always should keep in mind that one of the most conspicuous characteristics of the Poisson distribution is the assumption of absolute randomness of occurrence of an event. This conditional requirement most likely will never be met in actuality although admittantly it might be approached with low free flowing traffic volumes under ideal conditions (1, 14*). It is of great importance to the traffic engineer to know the extent to which he can rely upon such theoretical simulative representations.

Basically, there are two different concepts for collecting data of vehicle arrivals at an intersection. The first is to measure time headways or gaps which usually requires great efforts, exact preplanning and high expenditures but, in turn, yield excellent informative results. This normally is undertaken by highly sensitive electronic devices or detectors similar to those described by May (2, 101-107). Another excellent listing and review of such instruments currently in use in Germany is presented by Huber (3, 1-12). In this case, the time distances between successive vehicle arrivals are automatically measured and recorded, and it is apparent that practically all gaps or

---

* Figures in parentheses refer to listings in the bibliography on page 49 . The first number in parentheses which is underscored refers to the author's number in the bibliography. The second number, if any, refers to page numbers.

headways with fractional precision up to the nearest hundredth of a second crossing the observation point may occur (4, 562).

The second method, which is much easier and convenient, is to count vehicles arriving during specific time intervals. Hence, the traffic engineer sharply differentiates between a gap (continuous) and counting (discrete) distribution (5, IV-2,9). It is obvious that there is a great degree of correspondence between the two (6, 55). Their practical significance was mentioned above.

In this research it was expected that, because of substantial interference to traffic flow, theoretical discrete frequency distributions of arrivals from three different directions would by no means be an acceptable approximation to actual field observations. Hence it was intended to measure to some degree the magnitude of deviations between these two concepts. Primarily there were four reasons to believe there would be some distorted nonrandom results:

1. The majority of previous researchers had proved the above hypothesis true and were inclined to assume major interference.
2. Grabe (10, 15-20) lists several situations within the metropolitan area of Hannover, Germany where he found interference by traffic lights, tramway stops without islands, insufficient sight distance conditions, parking permitted on one or both sides

of the intersection approach legs, etc. His basic suggestion was to develop a scale for the various degrees of interference and the consideration of so called "Störfaktoren" (coefficients of interference) which he attempted to categorize according to the relation:

$$\mu_S = 1 + (\mu_0 - 1)\, e^{-cS} \qquad (1)$$

where

- S = distance from the source of interference (m)
- c = reduction factor $\left[\frac{1}{m}\right]$
- μ_0 = coefficient of interference immediately behind the source of interference
- μ_S = coefficient of interference at distance S (meters) from the source of interference

Figure 1 shows the asymptotical shape of this function.

3. Another scientific attempt taking into consideration the effect of various interferences was made by Woerner (14, 13). This is illustrated in Figures 2.1 and 2.2. Woerner explains his basic ideas upon the shape of his interference function:

> "[Ausnahmslos konnte festgestellt werden]: Grössere Häufigkeit kleiner und grosser Lücken und geringere Häufigkeit mittlerer Lücken. Mit anderen Worten: Die Zeitlückensummenlinie des gestörten Verkehrsstromes muss im Bereich kleiner λ unter, im Bereich grosser λ über der $P(x)_{und.}$-Kurve verlaufen."

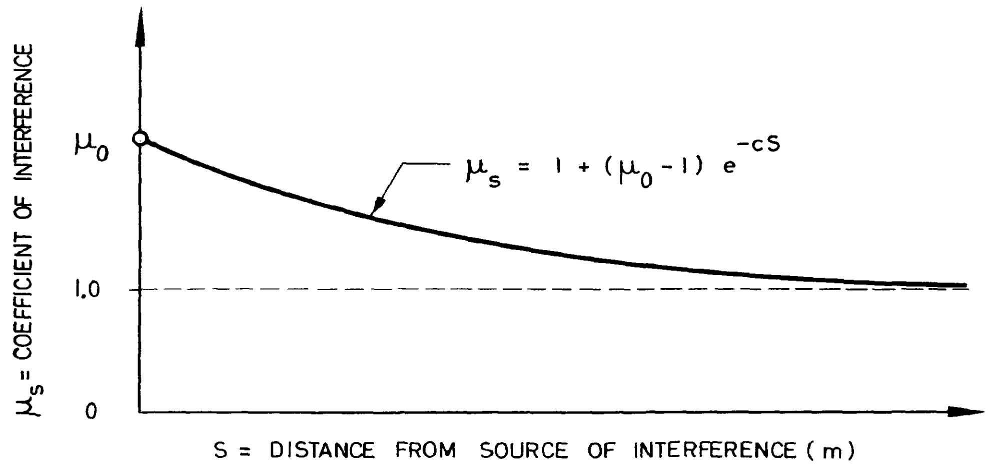

FIGURE 1. RELATION BETWEEN COEFFICIENT OF INTERFERENCE AND DISTANCE FROM SOURCE OF INTERFERENCE

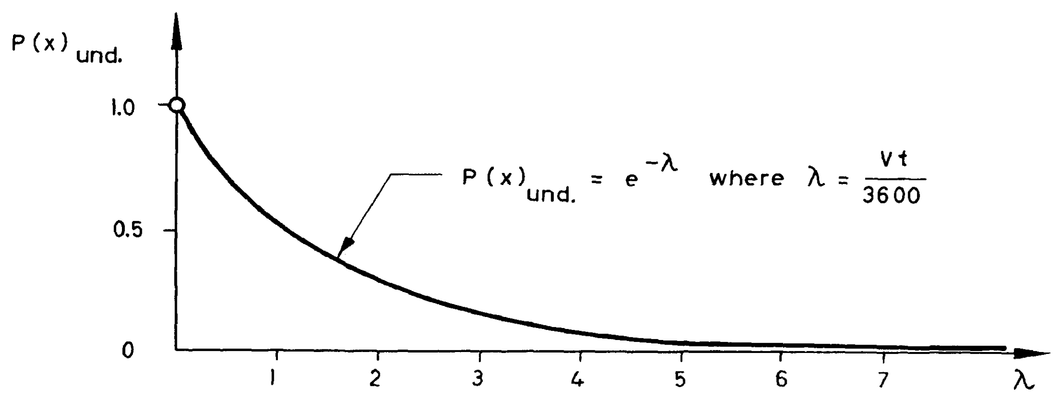

FIGURE 2.1 BASIC SHAPE OF THE FUNCTION $P(x) = e^{-\lambda}$ FOR UNDISTURBED TRAFFIC FLOW

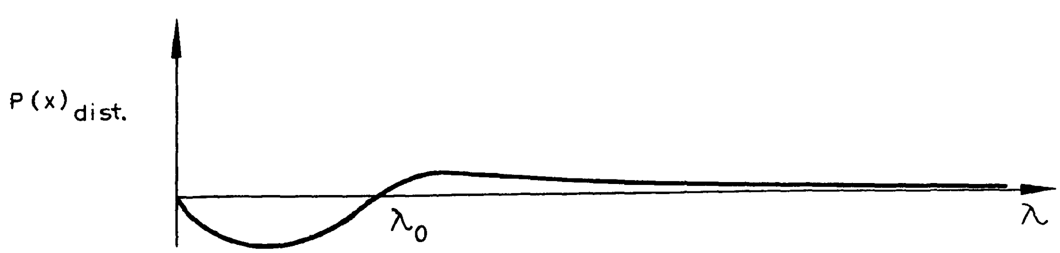

FIGURE 2.2 BASIC SHAPE OF AN INTERFERENCE FUNCTION

The factor λ is defined as

$$\lambda = \frac{Vt}{3600} \tag{2}$$

where

V = vehicles per hour (vph)

t = time interval (seconds)

4. Naturally one was tempted to agree with the above based only upon the expectation that interference elements in the very near vicinity of the intersection under investigation would effect traffic flow considerably.

B. Purpose of this Investigation and Objectives

Through motivation supplied by unexpected findings which convincingly indicated high levels of conformity with related discrete statistical models, the following objectives were investigated in detail:

1. Application of the Poisson distribution to the observed traffic situations to determine if the Poisson model sufficiently represents the observed arrival pattern using the χ^2 test.
2. Examine to fit either the binomial, negative binomial or generalized Poisson distribution as predetermined by the mean-variance relation criterion.
3. Comparison of the Chi-square results obtained under 2 with the corresponding results obtained by testing the Poisson distribution for "Goodness of Fit".

II. HISTORICAL BACKGROUND

Mathematics, probabilistic statistics, computerization, simulative analyses, and the science of traffic engineering are closely interrelated. Advanced traffic flow theories imply the knowledge and understanding of these tools. Consequently, it seems appropriate to attribute great merit and admiration towards the early developers of these calculative and estimative branches of natural science.

It was noticed that according to Fisher (11, 21-22) Thomas Bayes hesitantly published an essay in 1763 "...containing the first attempt to use the theory of probability as an instrument of inductive reasoning." In 1820, Laplace analyzed statistical populations and introduced their independent characteristic components, as the mean, variance, and other cumulants. It was no one other than the incomparable genius of Carl Friedrich Gauss who broadened and perfected statistical estimation considerably and also deserves credit for systematic fitting of regression formulae, simple and multiple, by the method of least squares.

The first modern test of significance goes back to Karl Pearson who in 1900 measured discrepancies between observations and hypotheses by a test known as Chi-square. In 1922, Prof. Pearson also published his "Tables of the Incomplete Γ Function" which is in use today for binomial exponential expansion series.

Jacob Bernoulli's name receives honorable mention in connection with the applications of the binomial distribution by Gerlough (12, 3), Huber (3, 24), and Schwar-Puy-Huarte (5, III-1). It has been recorded, that Ladislaus von Bortkiewicz made the first use of the Poisson distribution to treat statistical populations of death due to a "kick of a horse" among members of ten Prussian Cavalry Corps (12, 5) (9, 69)(13). Gerlough (12, 2), Miller (9, 69), May (2, 107), Woerner (14, 8), and Behr (ed.) (15, 178-182) credit William F. Adams, through his "Road Traffic Considered as a Random Series" in 1936, with making the first application of the Poisson distribution to vehicle arrivals at a point on a street. J.P. Kinzer made use of a binomial expansion in his theoretical thesis discussion on "Application of the Theory of Probability to Problems of Highway Traffic" in 1933. This work was reviewed by Lloyd F. Rader in 1934. These two investigators are regarded as Adam's immediate predecessors. It is interesting that Adams in his early pioneering work surmised "...that the Poisson distribution might be applicable for traffic volumes as high as 1,000 vehicles per hour; however, his data were for volumes up to 500 vph." (2, 107). Garwood made the first attempts to benefit from these findings by applying these theories mathematically to vehicular-controlled traffic signals in 1940.

Immediately after World War II, considerable effort was made by a group of researchers at the Bureau of Highway Traffic at Yale University to further test, refine, and supplement the previous results. Researchers like Greenshields, Normann, Raff, and Ricker contributed significantly in the 40's and 50's until, in 1954, Karl Moskowitz proceeded to test additional mathematical statistical theories (Garwood, Tanner) by measuring time headways (gap distribution) comprehensively in California. Moskowitz concluded: "The unfortunate distance (geographically speaking) between English brains and California traffic is remarked once again." (2, 107)

In 1955, the dual-published but entirely independent elaborations on "Poisson and Traffic" by Daniel L. Gerlough (12) and André Schuhl (7) won high recognition by traffic engineers. And finally, the names of Frank A. Haight (16, 101-105), a research mathematician, and Prof. Leo A. Goodman (17, 123-125) should receive honorable mention. They treated and analyzed discrete distributions mathematically and Goodman apparently discovered the generalized Poisson distribution by introducing adjustable parameters called λ and k as population estimators (4, 559-560).

A summary of references which includes the application of mathematical statistical frequency distributions to measured time headway distributions or closely related to fitting theoretical distributions to measured data is given in May's paper (2, 109). Ronald Aylmer Fisher (11, v) in his preface said "...it has become a matter of considerable difficulty for a research student to gain a correct idea of the present state of knowledge of a subject in which he himself is interested."

III. REVIEW OF LITERATURE

A. Definition of Terms

Some of the definitions pertaining to statistical distributions and vehicular flow problems in traffic engineering are shown in Table I. Most of the terminology listed has been collected from recognized authorities or publishers. The source, if any, is indicated in parentheses following the definition.

B. Previous Research

There is a considerable amount of published as well as unpublished literature concerning numerous attempts to fit mathematical statistical distributions to observed data. A proposed model can be verified only by comparing its predictions with actual observations. In the case of probability distribution models, the comparison is usually between the shape of the distribution curve and the shape of the plotted data. Publications cited in the following paragraphs are to a great extent directly related to the investigation of this problem.

Drew (25, Chapters 7 and 8) reduces the often highly complex functional mathematics of some distributions to restricted listings of their characteristic parameters and individual estimators. He gives the traffic engineer the valuable hint for attempts to fit a discrete distribution as follows:

TABLE I. TERMINOLOGY

No.	Symbol	Term	Definition
1.	h	Time headway	1. The time interval between passages of consecutive vehicles moving in the same lane (measured between corresponding points on the vehicles)(18, vii).
			2. Time separation of the leading edges of successive vehicles (19, 86).
			3. The interval in time between individual vehicles measured from head to head as they pass a given point (1, 16).
			4. ...measured from center to center* of successive vehicles (20, 170).
2.	g	Vehicular gap	1. The interval in time or distance between individual vehicles measured from the rear of one vehicle to the head of the following vehicle (1, 16).
			2. Gaps are normally measured from front-to-front of the successive vehicles and, thus, include the length of the lead vehicle* (21, 49).
3.	ℓ	Lag	1. Time interval measured from the arrival of a side-street vehicle at the stop bar of the intersection approach to the crossing of the path of this vehicle by the first main-street vehicle. Lag intervals are measured between the times when the fronts of the vehicles arrive at or cross their respective determination points (21, 49).

* Because of discrepancy, this terminology has been disregarded in this study.

Table I (continued)

No.	Symbol	Term		Definition
4.	s	Spacing	1.	The interval in distance from head to head of successive vehicles (1, 16).
			2.	The distance between consecutive vehicles moving in the same lane (measured between corresponding points on the vehicles)(18, viii).
5.	q	Flow	1.	[Flow] is measured by the number of vehicles passing a particular station during a given interval of time (22, 184).
			2.	The number of vehicles passing a point during a specified period of time; often referred to as "volume" when expressed in vehicles per hour measured over an hour (18, vii).
6.	p(x), P(x)	Probability	1.	The likelihood of occurrence of an event (18, vii).
7.	σ	Standard deviation	1.	A statistical measure of the dispersion of data from the mean (18, viii).
			2.	...measures average spread of curve about the arithmetic mean. Not used on open-ended curves (23, 106).
			3.	The root mean square (23, 5).
			4.	...is a measure of spread about the mean (5, I-3).
			5.	...is a statistical measure of the spread about the mean (24, V-5).

Table I (continued)

No.	Symbol	Term		Definition
8.	σ^2, Var	Variance	1.	The squared standard deviation of a statistical distribution.
9.	$\bar{x}$	Mean	1.	...represents the center of gravity of the distribution, that is, the "weight x distance" midpoint (23, 105).
			2.	...is a measure of the central tendency of the data (24, V-5).
10.	$s_{\bar{x}}$	Standard error of the mean	1.	This is a statistic that indicates the confidence with which the sample mean may be assumed to be the actual mean...[of a population](24, V-6).
11.	$s_{\bar{x}}^2$	Mean variance	1.	The squared standard error of the mean.
12.	χ^2	Chi-square	1.	Significance test for "Goodness of Fit".

> "The procedure for fitting a discrete distribution to observed data is to first compute the mean $\bar{x}$ and the variance s^2 of the observed data. If the mean and variance are approximately equal, the Poisson distribution can be used to compute the theoretical probabilities. However, if the variance is appreciably greater than the mean, the negative binomial can be used; if the variance is appreciably less than the mean, the binomial distribution can be used. In the last two cases the mean and variance of the observed data are equated to the first two moments of the desired distribution in order to estimate the parameters of the distribution."

This concept proved to be very helpful for the research study reported herein. Other possibilities were reported by Haight (4, 557-564)(16, 101-105) who successfully superimposed his ideas of the generalized Poisson distribution to seventeen observations of freeway traffic for the Shirley Highway near Los Angeles, California. From a nomograph which was developed under his guidance he obtained estimators for his parameters λ and k using the approximate formula:

$$\lambda = mk + \frac{1}{2}(k - 1) \qquad (3)$$

where

λ and k are distribution parameters
m is the sample mean

and where the reciprocal $\frac{1}{k}$ $(=\beta)$ is called by Haight the coefficient of randomness.

Haight refers to scientific derivations of his approach (16). He even refines his own elaborations to create the term "hyper-randomness" which would indicate a condition where values of β exceed unity. He states "...but there seems to be some evidence that a multi-lane freeway with very high traffic volume might be hyper-random." This, in fact, contradicts sharply with the original assumption that the highest degree of randomness occurs at very low traffic volumes.

It should further be noted, that the use of Haight's nomograph is, unfortunately but logically, limited to conditions where $\bar{x} \geq s^2$. This then seems to be synonymous with Drew's suggestion regarding the use of the binomial distribution.

Haight finally demonstrates the effect of fractional values of the parameter k by the use of tables of the incomplete Γ function (26), but admittantly remarks that the added accuracy is mainly theoretical in value.

Drew and Pinnell (8)(9, 76) were also eager to try the fit of the negative binomial and binomial distribution compared to the Poisson law, and they tested their results by Pearson's Chi-square distribution. They found a satisfactory conformity of observed data with Poisson during peak period arrivals. Two counting plans were used by Drew and Pinnell, one using five minute counting intervals, the other using one minute intervals. They also measured arrivals during off-peak periods and discovered certain implications of interest. In conclusion, they found sufficient support to assume that the negative binomial distribution produced a slight advantage over the Poisson distribution, and that only 7.5% of all forty cases studied were best approximated by the binomial model.

May (2, 101-136) presents detailed and comprehensive investigations in tabular and graphical form where he tried to observe relations between time headway distributions and occupancy levels, traffic volumes, and different locations. He analyzed central tendencies, cumulative frequencies, mode and median characteristics, platoon behaviour, and developed equations correspondingly. Unfortunately, May on the part of discrete distributions, did not investigate the negative binomial, binomial or generalized Poisson model; he summarizes under paragraphs 5 through 7:

> "5. There was significant difference between each measured time headway distribution and the Poisson distribution (negative exponential).
> 6. The use of larger time intervals and cumulative frequencies, and shift of the Poisson distribution, provide closer agreement between the measured and Poisson distribution. Even with these modifications, the Poisson distribution does not appear to be satisfactory for locations and traffic conditions studied.
> 7. The composite normal-Poisson distribution and the log-normal distribution show promise as being representative of measured time headway distributions."

Therefore, recapitulating in brief, remarkable research has been done so far, but it seems to be difficult at this point in the state of the art to find a common denominator for a generalized, widely applicable concept to use in this particular problem of traffic engineering.

IV. METHODOLOGY

A. Test Location and Procedures

The site chosen for this research was located within the business district of Rolla (population 14,000), the county seat of Phelps County, Missouri, at the intersection of Rolla and 11th Street. Field data were collected and observations recorded on Friday, July 7, 1972.

The intersection is of the four-leg right-angle type and is restricted on Rolla Street to one-way, two-lane traffic from North to South. Figures 3 through 5 illustrate the situation.

Traffic counts were conducted three times on the day stipulated above, namely in the morning from 7:30 a.m.-9:30 a.m., at noon from 11:30 a.m.-1:30 p.m., and during the afternoon peak hours from 3:30 p.m.-5:30 p.m. All vehicles were recorded at the time of their respective arrival at the "Property" lines of the intersection. Figure 6 illustrates the geometric features of the intersection in question and defines the property lines. Because of the one-way restriction on Rolla Street, no traffic arrived from the South.

When counting, no difference was made with respect to different vehicle types, but motorcycles, bicycles, and pedestrians were disregarded because of their incompatible traffic flow characteristics. Arrival times were manually recorded to the nearest five seconds by means of a stop

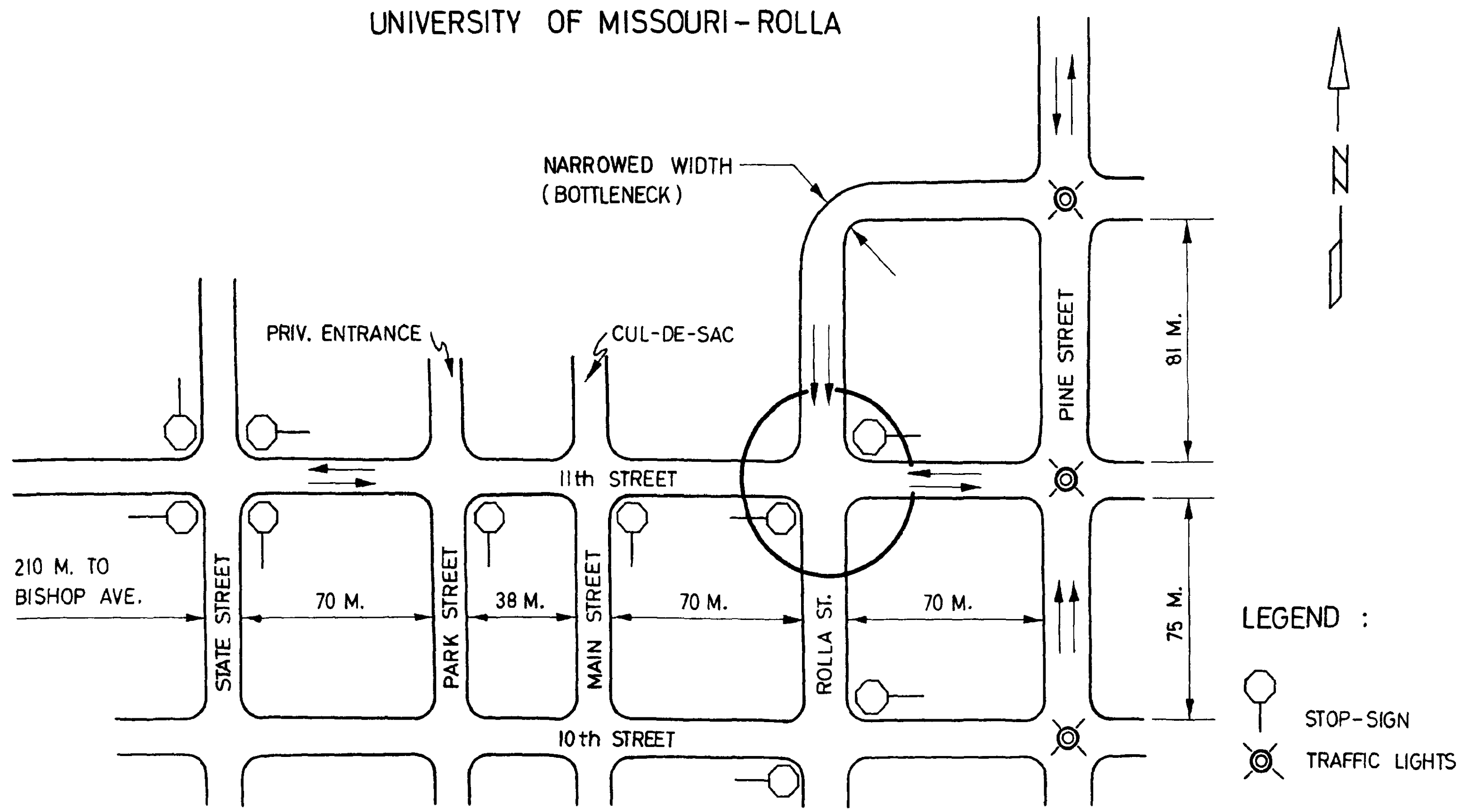

FIGURE 3. LOCATION OF INTERSECTION ROLLA AND 11th STREET IN ROLLA
NOT TO SCALE

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FIGURE 4. INTERSECTION ROLLA AND 11th STREET IN ROLLA, LOOKING NORTH

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FIGURE 5. INTERSECTION ROLLA AND 11th STREET, LOOKING EAST (THE TRAFFIC LIGHTS AT PINE AND 11th STREET ARE SEEN IN THE BACKGROUND)

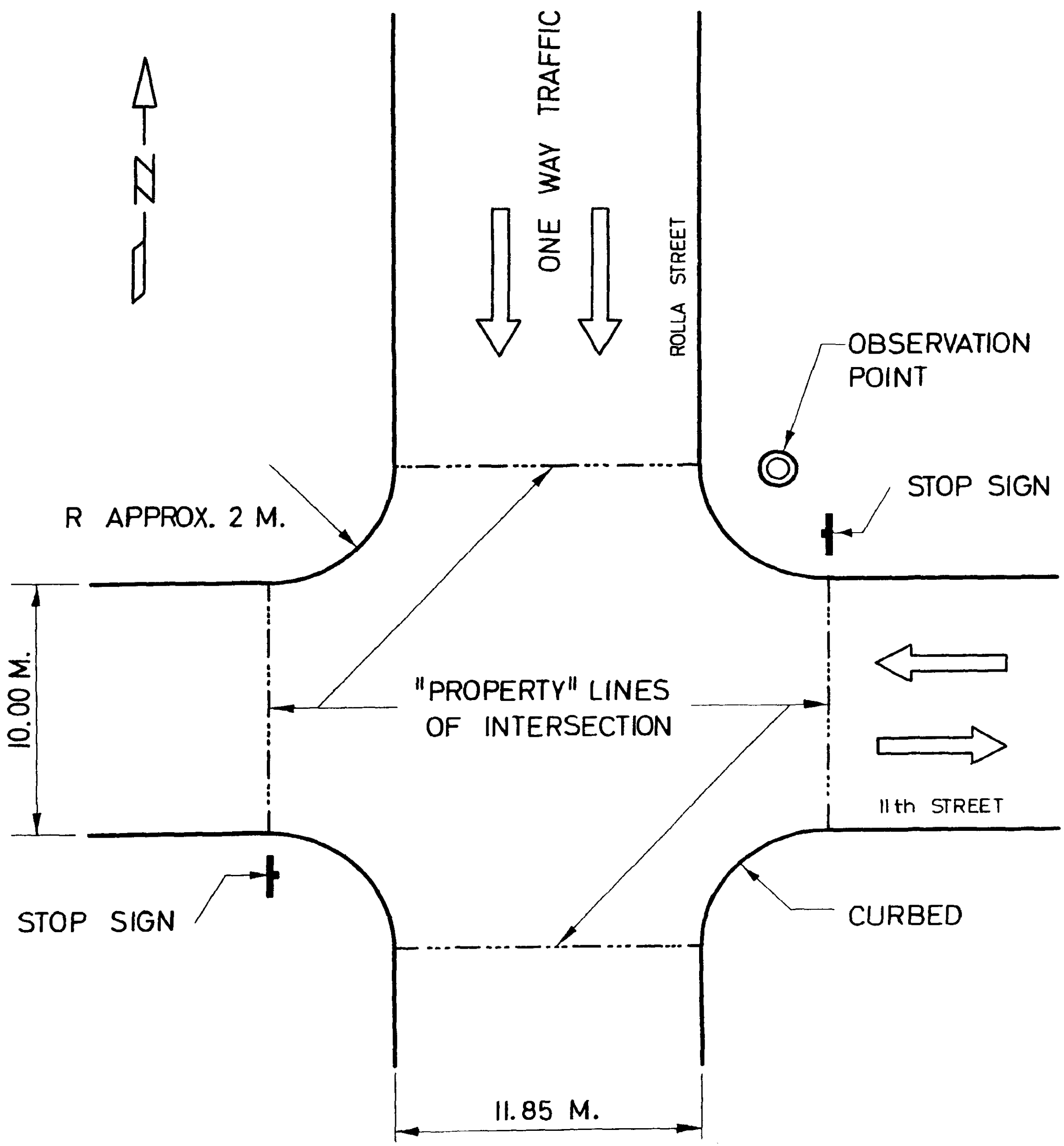

FIGURE 6. GEOMETRIC FEATURES AND "PROPERTY" LINES
NOT TO SCALE

watch.* Although this procedure was easy to follow throughout most of the study period, difficulties were encountered from 12:04 p.m. to 12:10 p.m. because of a very condensed arrival pattern during this period. However, a recount of traffic on Friday, July 14, 1972, at the same hours confirmed the observations with such a degree of conformity that it was felt safe to use them as representative values.

Traffic counts were started and terminated at exactly the given clock times independent of traffic flow. It is of importance to distinguish between this procedure and another type of detection in which the counting period starts immediately following the arrival of a vehicle. This technique is referred to as the synchronous case, whereas the first methodology is known as the asynchronous case (6, 55). In this study, as it seems to be logical for attempted applications of discrete frequency theories, the predetermined time intervals began precisely at half hours, thus representing the asynchronous case.

* A sample of these listing is attached in Appendix B, Table XI. Data on other approach directions and at other times is on file in the Department of Civil Engineering, University of Missouri-Rolla.

V. COMPUTATIONS

A. Introduction of the Interval (Gap) Probability Distribution

In order to demonstrate the conspicuous difference between the continuous (gap) and discrete (counting) distribution, it was necessary with regard to the continuous distribution to first group the data. The method used to record the data proved to be valuable for this manipulation. That is, the number of time headways observed was summarized for each of the 5, 10, 15, etc. seconds in difference of time distance. The theoretical equation was then computed according to the following formulae (27, Chapter III):

$$P(x) = \left(\frac{Vt}{3600}\right)^x \frac{e}{x!}^{-\frac{Vt}{3600}} \tag{4}$$

and

$$P(0) = e^{-\frac{Vt}{3600}} \tag{5}$$

where

V = hourly traffic volume (vehicles)

t = length of each observation (seconds)

From the Poisson distribution it is known that

$$\frac{Vt}{3600} = m \tag{6}$$

Hence, if we set

$$m = \frac{t}{T} \tag{7}$$

then

T = the mean of the interval (gap) probability distribution and the probability of a gap equal to or greater than t is given by

$$P(g \geq t) = e^{-\frac{t}{T}} \qquad (8)$$

From the above relationships it may be seen that (under conditions of random flow) the number of gaps greater than any given value will be distributed according to an exponential curve. Though most correctly described by a "negative exponential", this is also known simply as exponential distribution.

From Figures 7 through 9 it is obvious that no similarity exists in all of these results and that a complete and separate study could be undertaken to search for a satisfactory fit in the form of a Pearson Type III, Erlang, or gamma distribution by introducing appropriate adjustable parameters. The reader's attention is called to the fact that the plot of observed data for a continuous distribution should correctly be a smooth curve. Whereas the illustration for observed data of a discrete distribution should result in the typical step shaped histogram since it is not possible to register fractions of occurrences per a specified number of cars arriving during a time interval.

B. Application of Discrete (Counting) Probability Distributions

1. The Poisson Distribution

To prepare the data for the test of approximation to discrete frequency models, it was first necessary to predetermine suitable time intervals. This was done arbitrarily, but under consideration of the apparent difference in volumes. It was decided to assign intervals of 30 and 60 seconds to the North approach, 60 and 120 seconds to the West, and 120 seconds only to vehicles arriving from the East direction. The reason for selecting different time intervals was primarily because of the difference in vehicle arrivals per two-hour period and their particular sensitivity for comparison with discrete distributions. Thus, it was felt that the shorter time intervals should be applied to the direction with the higher volumes and the 120 seconds interval should be an appropriate value for the considerably lower traffic volume approaching the intersection from the East. In Appendix B, Table XI, the number of vehicles arriving was counted and listed as observed frequencies in tabular form.

The Poisson probability function was then applied to all individual situations, that is, if $P(n|qT)$ is the probability of n arrivals in T seconds and q is the traffic flow in vehicles per hour (see Figures 10 through 24, where $(qT) = \mu$), then

$$P(n|qT) = \frac{(qT)^n e^{-qT}}{n!} \tag{9}$$

where

$n! = n(n-1)(n-2)(n-3)\ldots3\cdot2\cdot1$

qT = the mean and variance of the theoretical Poisson distribution

The computations may be performed in a variety of ways. Values of e^{-qT}, sometimes defined as e^{-m} or Poisson terms (27) where

$$m = \frac{\text{total number of vehicles per hour}}{\text{total number of time intervals per hour}}$$

can be obtained from tables of appropriate publications (28)(29). In this study, computations were processed by electronic calculators and the following simplifications, known as "recursion formulae", proved to be very convenient:

$$P(0) = e^{-qT} \tag{10}$$

$$\frac{P(n)}{P(n-1)} = \frac{\dfrac{(qT)^n}{n!} e^{-qT}}{\dfrac{(qT)^{(n-1)}}{(n-1)!} e^{-qT}} \tag{11}$$

From (11)

$$P(n) = \frac{(qT)}{n} P(n-1) \qquad x \geq 1 \tag{12}$$

Thus, from (10) and (12) it follows that

$$P(0) = e^{-qt}$$

$$P(1) = \frac{(qT)}{1} P(0)$$

$$P(2) = \frac{(qT)}{2} P(1)$$

$$P(3) = \frac{(qT)}{3} P(2)$$

$$P(4) = \frac{(qT)}{4} P(3), \text{ etc.}$$

This can be expressed in general form:

$$P(x + 1) = P(x) \frac{m}{(x + 1)} \tag{13}$$

Table II shows an example of processing these computations.

2. <u>The Chi-Square Test</u>

The results obtained from the above computations were tested against observed frequencies by Karl Pearson's Chi-square test. The expression of the probability function of the χ^2 distribution is

$$f(\chi^2) = C(\chi^2)^{\frac{\nu - 2}{2}} e^{-\frac{\chi^2}{2}} \tag{14}$$

where

ν = the number of degrees of freedom

C = a constant assigned such that the following condition is guaranteed

TABLE II. PREDICTION OF POISSON ARRIVALS AT ROLLA/11th STREET BY THE METHOD OF "RECURSION FORMULAE"

NORTH APPROACH, 11:30 a.m.-1:30 p.m.

Date: July 7, 1972 Time Interval: 30 sec.

Basic data: V = hourly volume = 294 vph

t = length of time interval = 30 seconds

$$\mu = \frac{Vt}{3600} = \frac{294 \cdot 30}{3600} = \underline{2.45}$$

$$e^{-\mu} = e^{-2.45} = \underline{0.08629}$$

$$n = \frac{3600}{t} = \frac{3600}{30} = \underline{120}$$

Theoretical Poisson Frequency	Poisson Term	Computation	Result (Number of Occurrence)
F	$n\,(\frac{1}{0!})\,\mu^0 e^{-\mu}$	(120)(1)(1)(0.086)	20.71
F_1	$F_0\,(\frac{m}{1})$	(20.71)(2.45)	50.74
F_2	$F_1\,(\frac{m}{2})$	(50.74)(1.23)	62.16
F_3	$F_2\,(\frac{m}{3})$	(62.16)(0.82)	50.76
F_4	$F_3\,(\frac{m}{4})$	(50.76)(0.61)	31.09
F_5	$F_4\,(\frac{m}{5})$	(31.09)(0.49)	15.23
F_6	$F_5\,(\frac{m}{6})$	(15.23)(0.41)	6.22
F_7	$F_6\,(\frac{m}{7})$	(6.22)(0.35)	2.18*
F_8	$F_7\,(\frac{m}{8})$	(2.18)(0.31)	0.67*
F_9	$F_8\,(\frac{m}{9})$	(0.67)(0.27)	0.18*

* Theoretical frequencies < 5 not suitable for Pearson's χ^2 - test (12, 10)(27, 14, 117).

$$\int_{\alpha}^{\beta} f(x)dx = 1 \tag{15}$$

Schwar and Puy-Huarte demonstrate the relation between (χ^2) and various degrees of freedom (ν) in "Statistical Methods in Traffic Engineering" (5, VI-2). During the process of these computations it was found that it was of eminent importance how the data to be compared was grouped in order "...to meet the requirement of the χ^2 test that the theoretical frequency be at least 5 in any group." (12, 10)(27, 14, 117) By not conforming to this condition it was observed that the results were distorted and unrepresentative to such an extent that a fruitful conclusive judgment was impossible. Chi-square results were obtained from the following relationship:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E)^2}{E} \tag{16}$$

where

O_i = observed values

E = theoretical expected values

For the case of a binomial distribution, according to Griffith (23, 345), this can be represented as

$$\chi^2 = \sum_{i=1}^{n} \frac{(p_i - np)^2}{npq} \tag{17}$$

or a ratio of n times the sum of squares of the deviations of each observed value from the mean divided by the variance, that is, ns^2/σ^2, which is a chi-square statistic with ν degrees of freedom. Table III shows a typical example of this process. See paragraph V.C. for application of the χ^2 test.

After completion of these computations, the mean and variance of each observed population were computed in order to establish the platform for Drew's (25, 130) and Gerlough's (27, 21) mean-variance interrelation requirement. The mean was computed using the formula:

$$\bar{x} = \frac{\sum_{i=0}^{n} f_i x_i}{n} \tag{18}$$

where

$\bar{x}$ = arithmetic mean of the population

Σ = sigma - a statistical symbol meaning "the sum of"

$\Sigma f_i x_i$ = sum of the mean frequencies

n = total number of vehicles observed

For the variance, the following equation was used:

$$s^2 = \frac{\sum_{i=1}^{n} f_i (x_i)^2 - \frac{\left(\sum_{i=1}^{n} f_i x_i\right)^2}{n}}{n - 1} \tag{19}$$

where

s^2 = sample variance

$\Sigma f_i (x_i)^2$ = sum of the mean square frequencies

$(\Sigma f_i x_i)^2$ = square of the sum of the mean frequencies

TABLE III. χ^2 TEST OF A POISSON FREQUENCY DISTRIBUTION AGAINST OBSERVED DATA OF VEHICLE ARRIVALS AT ROLLA/11th STREET IN ROLLA

NORTH APPROACH, 11:30 a.m.-1:30 p.m.

Date: July 7, 1972 Time Interval: 30 sec.

Number of Vehicles Arriving per Time Interval (x_i)	Observed Frequency (f_i)	Postulated (Theoretical) Frequency (F_i)	Computation $\frac{f_i^2}{F_i}$
0	22	20.71	23.37
1	58	50.74	66.30
2	58	62.16	54.12
3	42	50.76	34.75
4	35	31.09	39.40
5	14	15.23	12.87
6	4	6.22	2.57
7	3 }	2.18 }	
8	3 } 7*	0.67 } 3.08*	15.90
9	1 }	0.18 }	
Totals	n = 240	240.00	$\left(\sum \frac{f_i^2}{F_i}\right) = 249.27$

$$\chi^2 = \left(\sum \frac{f_i^2}{F_i}\right) - n = 249.27 - 240.00 = \underline{9.27}$$

* The brackets indicate grouping for the χ^2 test.

TABLE IV. DATA ANALYSIS OF VEHICLE ARRIVALS AT ROLLA/11th STREET IN ROLLA NORTH APPROACH, 11:30 a.m.-1:30 p.m.

Date: July 7, 1972 Time Interval: 30 sec.

No.	Arrivals per Interval	Frequency (Number Observed)	Computations	
	x_i	f_i	$f_i\ x_i$	$f_i(x_i)^2$
(1)	(2)	(3)	(4) = (2) x (3)	(4) x (2)
1	0	22	0	0
2	1	58	58	58
3	2	58	116	232
4	3	42	126	378
5	4	35	140	560
6	5	14	70	350
7	6	4	24	144
8	7	3	21	147
9	8	3	24	192
10	9	1	9	81
Totals		n = 240	Σ = 588	Σ = 2142

$$\bar{x} = \frac{\Sigma f_i x_i}{n} = \frac{588}{240} = \underline{2.45 \text{ vehicle arrivals per interval}}$$

$$s^2 = \frac{\Sigma f_i(x_i)^2 - \frac{(\Sigma f_i x_i)^2}{n}}{n-1} = \frac{2142 - \frac{(588)^2}{240}}{240 - 1} = \frac{701.4}{239} = \underline{2.9347};$$

$$s = \sqrt{s^2} = \sqrt{2.9347} = 1.7131; \quad s_{\bar{x}}^2 = \frac{s^2}{n} = \frac{2.9347}{240} = \underline{0.0122};$$

$$s_{\bar{x}} = \sqrt{s_{\bar{x}}^2} = \sqrt{0.0122} = 0.1106;$$

where s = standard deviation of the distribution

$s_{\bar{x}}^2$ = mean variance

$s_{\bar{x}}$ = standard error of the mean.

A typical example of these computations is shown in Table IV.

Using the ratio of the mean divided by the variance, either the binomial or negative binomial distribution was chosen to test whether it would be possible to produce a better approximation than the Poisson distribution had provided. Refer to Chapter III.B for explanations. Results of the above check revealed that in nine cases the variance was greater than the mean (negative binomial), in five cases the variance was smaller (binomial), and in one situation (West approach, 7:30 a.m.-9:30 a.m., 120 seconds time interval) the difference was so small that the application of the binomial distribution with a computed 768th power value would have been meaningless. In this case, the similarity between the Poisson and binomial curve would not show a significant difference. Here it was apparent that the Poisson distribution in fact is an approximation to the binomial distribution, primarily for situations when (x) is large and (p) is small. The two concepts approach each other when $x \to \infty$ and $n \to 0$.

3. Application of the Negative Binomial Distribution

By setting the number of vehicles arriving during a specified interval of time $x = (n - k)$ where the term $(n - k)$ can be defined as the probability of k positive results or successes during n occurrences of arrivals, the probability of x vehicles arriving per time period is given by the negative binomial or Pascal distribution which may be written in the form

$$P(x) = \binom{x + k - 1}{k - 1} p^k (1 - p)^x \tag{20}$$

or

$$P(x) = \frac{(x + k - 1)!}{(k - 1)![(x + k - 1)-(k - 1)]!} p^k (1 - p)^x \tag{21}$$

Substituting for $(1 - p) = q$ and reducing, we obtain

$$P(x) = \frac{(x + k - 1)!}{x!(k - 1)!} p^k q^x \tag{22}$$

where

x = number of vehicles arriving per specified time interval

p = mean/variance ratio $\left(\frac{\bar{x}}{s^2}\right)$

$k = \frac{\bar{x}}{(s^2 - \bar{x})}$

It should be mentioned that upon computing the parameter k fractional results were obtained, and that the evaluation of fractional values of factorial series is cumbersome and time consuming. Theoretically, this can be done by using tables of the normalized incomplete gamma function (26). For this investigation, however, k values were rounded to the nearest integer for convenience and because of insignificancy.

The previous described procedure of fitting theoretical distributions is also known as the "Method of Moments" (27, 10). This terminology indicates the fact that by computing sample moments (arithmetic mean, variance, etc.) one estimates the moments or parameters of the desired theoretical distribution.

Therefore, in order to add more information, the following relations are given for the negative binomial distribution:

$$\bar{x} = \frac{k(1 - p)}{p} \tag{23}$$

and

$$s^2 = \frac{k(1 - p)}{p^2} \tag{24}$$

4. Application of the Binomial Distribution

As listed on page 34, five observed situations offered an alternative to the postulated theoretical Poisson frequencies in the form of the binomial or Bernoulli distribution. This particular model originates in the simple realization that a set of repeated independent trials whose outcomes consist of only two mutually exclusive events and whose probabilities remain the same throughout the trials can be represented by the form $(p + q)^n$. By repeated multiplication of $(p + q)$ by itself one obtains the basic equation for the binomial family, namely:

$$(p + q)^n = \sum_{x=0}^{n} \frac{n!}{x!(n - x)!} p^x q^{(n-x)} \tag{25}$$

This yields the probability function as

$$P(x) = \frac{n!}{x!(n - x)!} p^x q^{(n-x)} \tag{26}$$

or written in another form:

$$P(x) = \binom{n}{x} p^x (1 - p)^{(n-x)} \tag{27}$$

where the distribution parameters p and n are estimated by the following equations

$$p = \frac{(\bar{x} - s^2)}{\bar{x}} \tag{28}$$

and

$$n = \frac{\bar{x}}{(\bar{x} - s^2)} \tag{29}$$

It should be noted that the summation of the term (p + q) in (25) above must always be equal to one. The expansion of the binomial term is sometimes given in the form (<u>5</u>, III-3):

$$1 = (q + p)^n = \binom{n}{0} q^n + \binom{n}{1} q^{(n-1)}$$

$$p \ldots \binom{n}{k} q^{(n-k)} p^k + \ldots \binom{n}{n} p^n \tag{30}$$

For the binomial distribution the statistical moments are defined as follows:

The mean ($\bar{x}$) = np

The variance (s^2) = np(1 - p)

The standard deviation (s) = $\sqrt{npq}$

As explained on page 35, here again only integers were used for values of the parameter n, although, as might be expected, fractional results were obtained (29). Again, the probability of 0, 1, 2, 3 ... cars arriving within a specified time interval was then computed.

C. The χ^2 Test for "Goodness of Fit"

Both the theoretical frequencies obtained by the negative binomial and binomial distribution then were tested against observed values. χ^2 results were obtained from:

$$\chi^2 = \left(\sum_{i=1}^{g} \frac{f_i^{\,2}}{F_i^{\,2}} \right) - n \qquad (31)$$

where

f = observed frequency for any interval or group

F = computed theoretical frequency for the same group

g = number of groups

n = total number of observations

The derivation of (31) is shown in (27, 116). The computed χ^2 results then were compared with χ^2 values from tables (30, 188-189). The number of degrees of freedom, ν, for discrete frequency distributions may be expressed as:

$$\nu = (g - 1) - A \tag{32}$$

where

g = number of groups

A = number of parameters estimated in the fitting process

For the reader's convenience, degrees of freedom for the most common discrete distributions have been listed in Table V. Accordingly, for the Poisson, negative binomial, and binomial distribution (ν) has been determined as $(g - 2)$, $(g - 3)$, and $(g - 3)$, respectively.

TABLE V. DEGREES OF FREEDOM

FOR DISCRETE FREQUENCY DISTRIBUTIONS

$$\nu = (g - 1) - A$$

No.	Distribution	A	Degrees of Freedom
1	Poisson	1	g - 2
2	Negative Binomial	2	g - 3
3	Generalized Poisson	2	g - 3
4	Binomial	2	g - 3

where

g = number of groups compared

A = number of parameters estimated in the fitting process

VI. RESULTS AND EVALUATION

Results of the "Goodness of Fit" test for the various discrete arrival distributions are shown in Tables VI through VIII. With regard to determining acceptance levels, the researcher or traffic engineer will predetermine confidence limits based upon his own experience and the purpose of his study. Drew (25, 125) and Gerlough (27, 33) suggest the use of preselected significance levels of $\alpha = 0.05$ (5%), although the latter also states that α values of 0.01 (1%) and 0.10 (10%) are popular. Since different investigators definitely will predetermine different acceptance levels, values for $\chi^2_{tab.}$ have been listed in Tables VI through VIII for $\alpha = 0.05$, $\alpha = 0.10$, and $\alpha = 0.25$ for convenience.

It was found that at the 5% significance level no acceptable fit could be produced for the East approach, 11:30 a.m.-1:30 p.m., 120 seconds time interval. At the $\alpha = 10\%$ significance level, in addition to the beforementioned situation, the fit of the Poisson distribution for the North approach during the noon peak period at 60 seconds time intervals did not produce a sufficient degree of conformity. Here again, traffic flow seemed to be interfered with by an upstream signalized intersection and a bottleneck or restricted one-lane right angle curve. It was apparent that these elements of disturbance had a more pronounced influence on the 60 seconds time interval than

TABLE VI. COMPARISON OF TESTS ON VARIOUS DISCRETE ARRIVAL DISTRIBUTIONS

Time: 7:30 a.m.-9:30 a.m.

Approach Direction	Time Interval (sec.)	Statistical Distribution	Mean ($\bar{x}$)	Variance (s^2)	Degrees of Freedom (ν)	χ^2 calc.	χ^2 tab. (α=0.05)	χ^2 tab. (α=0.10)	χ^2 tab. (α=0.25)
North	30	Poisson	1.43	1.43	4	0.906	9.488	7.779	5.385
		Negative Binomial	1.43	1.55	3	0.880	7.815	6.251	4.108
	60	Poisson	2.87	2.87	6	10.947	12.592	10.645*	7.841*
		Negative Binomial	2.87	3.60	5	7.597	11.071	9.236	6.626*
West	60	Poisson	1.04	1.04	3	0.244	7.815	6.251	4.108
		Binomial	1.04	0.96	2	0.038	5.991	4.605	2.773
	120	Poisson	2.08	2.08	4	6.682	9.488	7.779	5.385*
		Binomial	2.083	2.078		not elaborated			
East	120	Poisson	1.38	1.38	3	3.368	7.815	6.251	4.108
		Negative Binomial	1.38	1.63	2	3.744	5.991	4.605	2.773*

* Indicates significance

TABLE VII. COMPARISON OF TESTS ON VARIOUS DISCRETE ARRIVAL DISTRIBUTIONS

Time: 11:30 a.m.-1:30 p.m.

Approach Direction	Time Interval (sec.)	Statistical Distribution	Mean ($\bar{x}$)	Variance (s^2)	Degrees of Freedom (ν)	χ^2 calc.	χ^2 tab. (α=0.05)	χ^2 tab. (α=0.10)	χ^2 tab. (α=0.25)
North	30	Poisson	2.45	2.45	6	9.272	12.592	10.645	7.841*
		Negative Binomial	2.45	2.93	5	5.687	11.071	9.236	6.626
	60	Poisson	4.90	4.90	9	16.333	16.919	14.684*	11.389*
		Negative Binomial	4.90	8.29	8	9.438	15.507	13.362	10.219
West	60	Poisson	1.78	1.78	4	0.746	9.488	7.779	5.385
		Binomial	1.78	1.63	3	0.273	7.815	6.251	4.108
	120	Poisson	3.57	3.57	5	3.585	11.071	9.236	6.626
		Binomial	3.57	3.47	4	3.681	9.488	7.779	5.385
East	120	Poisson	3.78	3.78	6	13.754	12.592*	10.645*	7.841*
		Binomial	3.78	3.29	5	13.119	11.071*	9.236*	6.626*

* Indicates significance

TABLE VIII. COMPARISON OF TESTS ON VARIOUS DISCRETE ARRIVAL DISTRIBUTIONS

Time: 3:30 p.m.-5:30 p.m.

Approach Direction	Time Interval (sec.)	Statistical Distribution	Mean ($\bar{x}$)	Variance (s^2)	Degrees of Freedom (ν)	χ^2 calc.	χ^2 tab. (α=0.05)	χ^2 tab. (α=0.10)	χ^2 tab. (α=0.25)
North	30	Poisson	2.19	2.19	6	5.365	12.592	10.645	7.841
		Negative Binomial	2.19	2.68	5	2.318	11.071	9.236	6.626
	60	Poisson	4.38	4.38	8	5.584	15.507	13.362	10.219
		Negative Binomial	4.38	5.24	7	4.051	14.067	12.017	9.037
West	60	Poisson	1.62	1.62	4	1.186	9.488	7.779	5.385
		Binomial	1.62	1.47	3	1.067	7.815	6.251	4.108
	120	Poisson	3.23	3.23	5	4.605	11.071	9.236	6.626
		Negative Binomial	3.23	3.30	4	4.456	9.488	7.779	5.385
East	120	Poisson	3.18	3.18	5	4.118	11.071	9.236	6.626
		Negative Binomial	3.18	3.91	4	3.660	9.488	7.779	5.385

on the 30 seconds interval. However, the Poisson distribution also failed at the 30 seconds interval, but only at the $\alpha = 0.25$ significance level.

Vehicle arrivals of all North approaching traffic was best represented by the negative binomial distribution. A similar agreement was observed for the 60 seconds time interval for the West approach and the binomial fit. Furthermore, sufficient evidence was obtained to show that of all approach directions, traffic arriving from the West produced the closest approach to randomness, that is, the only case of rejection of the acceptance hypothesis occurred for the Poisson approximation at the 25% level. This could be expected, since there was no interference by traffic lights on this approach. The STOP signed intersections on the West approach leg did not effect the high degree of randomness significantly.

With respect to time periods, it was discovered that all theoretical distributions were acceptable for the afternoon time period (3:30 p.m.-5:30 p.m.) and this even at significance levels as high as 25%. This is substantiated by the somewhat relaxing atmosphere of traffic flow in contrast to the hectic, nervous pattern at earlier hours during the day.

VII. CONCLUSIONS AND RECOMMENDATIONS

The conclusions drawn from the results of this research are based upon the analysis of data from vehicular arrivals at the intersection of Rolla and 11th Street in Rolla, Missouri. It is believed that these conclusions are basically applicable to other urban intersections in smaller cities with general features similar to those at the location investigated.

A. Conclusions

1. Vehicular arrivals at an urban intersection can be reasonably well represented by discrete statistical counting distributions when the traffic flow is in the range of 200 to 400 vehicles per hour even when the traffic flow is disturbed to some extent by upstream regulative devices or a narrowed street width. Some effect caused by these interferences will be noticable.
2. For a rough and preliminary approximation, the Poisson distribution may be applicable with sufficient promise of providing an acceptable fit to observed data.
3. The various statistical distributions provide a better similarity to observed data during off-peak periods in a less hectic situation.

4. For more precise results, in the majority of the cases, better representation can be obtained by replacement of the Poisson model by the binomial, negative binomial or generalized Poisson distribution. The "Method of Moments" together with the mean/variance criterion offer a promising approach for selecting the frequency model most suitable.

B. Recommendations

1. There should be an investigation to determine why the effect of disturbance was relatively small. Attention should be given to the percentage of total vehicles which pass the upstream intersections without major delay.
2. The influence of further extending the arbitrarily preselected time intervals should be studied, particularly in relation to the time cycle of the traffic lights located on approach legs of the intersection under investigation.
3. Because of the sensitivity of the χ^2 test for small values and the understandable reluctance of researchers to submit such data to this test, it is recommended that better guidelines for its use should be developed by the establishment of certain limits based possibly

on other criteria than the "at least five in any group" suggestion.

4. Although the very helpful mean/variance criterion makes the choice between the binomial and negative binomial distribution relatively easy, no indication could be found in the literature as to what criteria should be used to decide between the binomial and generalized Poisson distribution which have the mean > variance condition in common.

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VITA

Horst Walter Kaminsky was born December 21, 1936, in Wanne-Eickel, West Germany. He received his primary and secondary education in Herne, Germany. In 1954, he graduated from the Pestalozzi Gymnasium (now Otto Hahn Gymnasium) and entered mason apprenticeship. Simultaneously, from 1954-56, he attended the Vocational School in Herne, class for mason handicraft. He received the certificate as mason journeyman and widened his practical experience as floor tiler and paver and reinforcement bender in order to fulfill part of the admission requirements for engineering colleges in Germany.

The degree of Tiefbauingenieur (grad.)(Bachelor of Science in Civil Engineering) was conferred on Mr. Kaminsky by the Staatliche Ingenieurakademie für Bauwesen in Holzminden, Germany, in 1959. Upon graduation he continued his studies at the Staatsbauschule München-Akademie für Bautechnik where he graduated with the degree of Hochbauingenieur (grad.)(Bachelor of Science in Architecture) in 1961.

From 1961-64, Mr. Kaminsky worked as an architect, structural designer, and construction supervisor with Carl F. Raue, Architects, Munich, Germany. In August 1964 he left Germany and was assigned Resident Engineer for a combined road and irrigation project by the firm of Dr. Ing. H. Müller, Consulting Engineer, Windhoek, South West Africa.

Mr. Kaminsky returned to Germany briefly in 1967 after completion of his contract in Africa and joined the staff of Lorenz-Bau GmbH, Special Foundation Enterprise, Iserlohn, Germany. With this company he held the position of General Superintendent in charge of pile and special foundations in Mid-Thailand from 1967-68.

After completion of this appointment he was employed as a Highway Design Engineer and Civil Engineer by Sverdrup & Parcel International, Architects-Consulting Engineers, St. Louis, Missouri, in their Southeast Asia regional office in Bangkok, Thailand.

When admitted to the Graduate School of the University of Missouri-Rolla in August 1971, Mr. Kaminsky moved directly from Thailand to the United States.

Upon completion of his graduate program at Rolla, Mr. Kaminsky will begin studies toward the Doctor of Philosophy degree at the University of California, Berkeley, majoring in transportation engineering.

Mr. Kaminsky is a Member of the American Society of Civil Engineers (ASCE), Personal Member of the Verein Deutscher Ingenieure (VDI)(Society of German Engineers), Associate Member of the Engineering Institute of Thailand under the Patronage of H.M. the King, Member of the Southeast Asian Society for Soil Engineering (SEASSE) and Member of the International Society for Soil Mechanics and Foundation Engineering (ISSMFE).

APPENDIX A

(Figures 7 through 24)

Time Headway Frequency Distributions

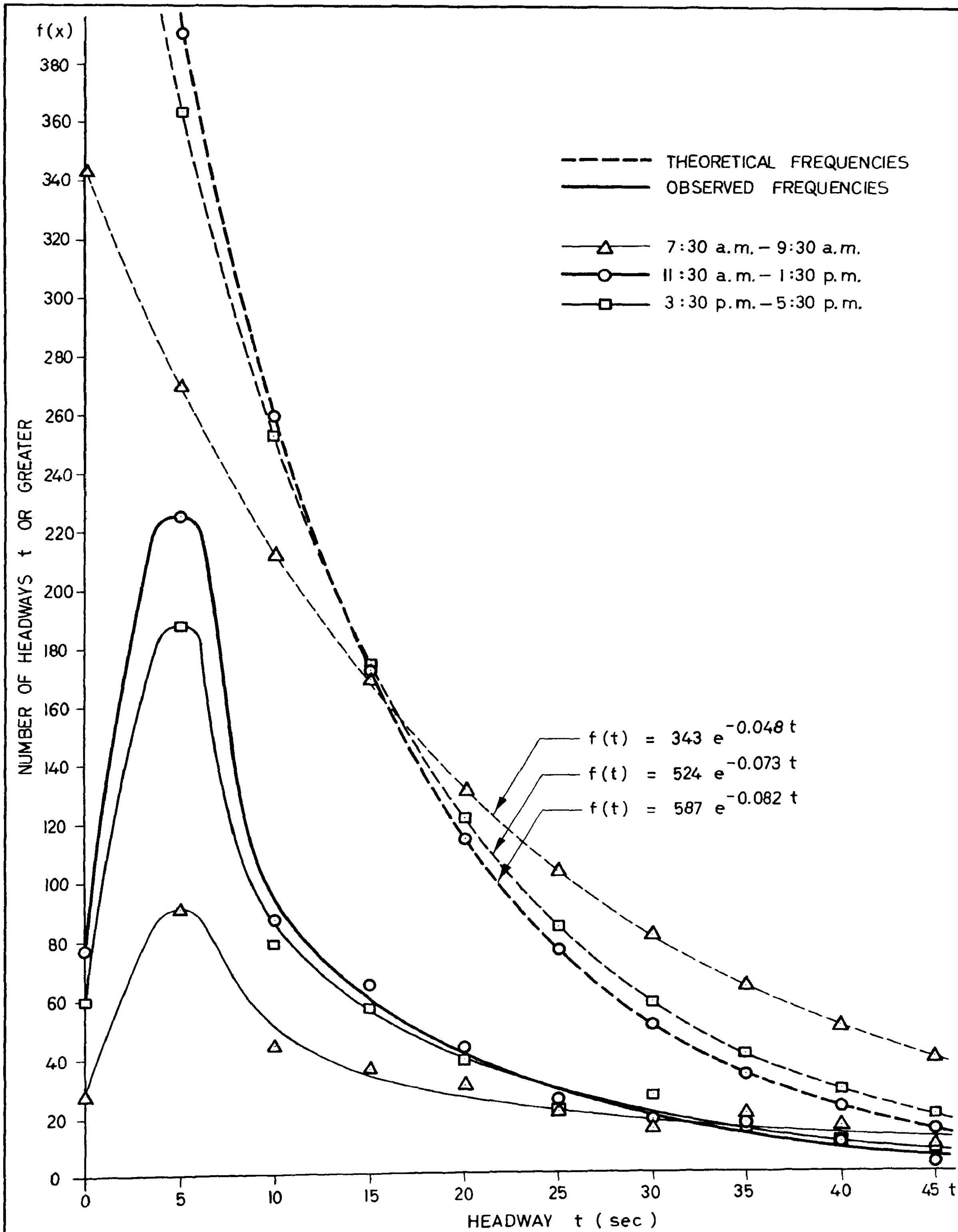

FIGURE 7. NEGATIVE EXPONENTIAL TIME HEADWAY DISTRIBUTION
NORTH APPROACH

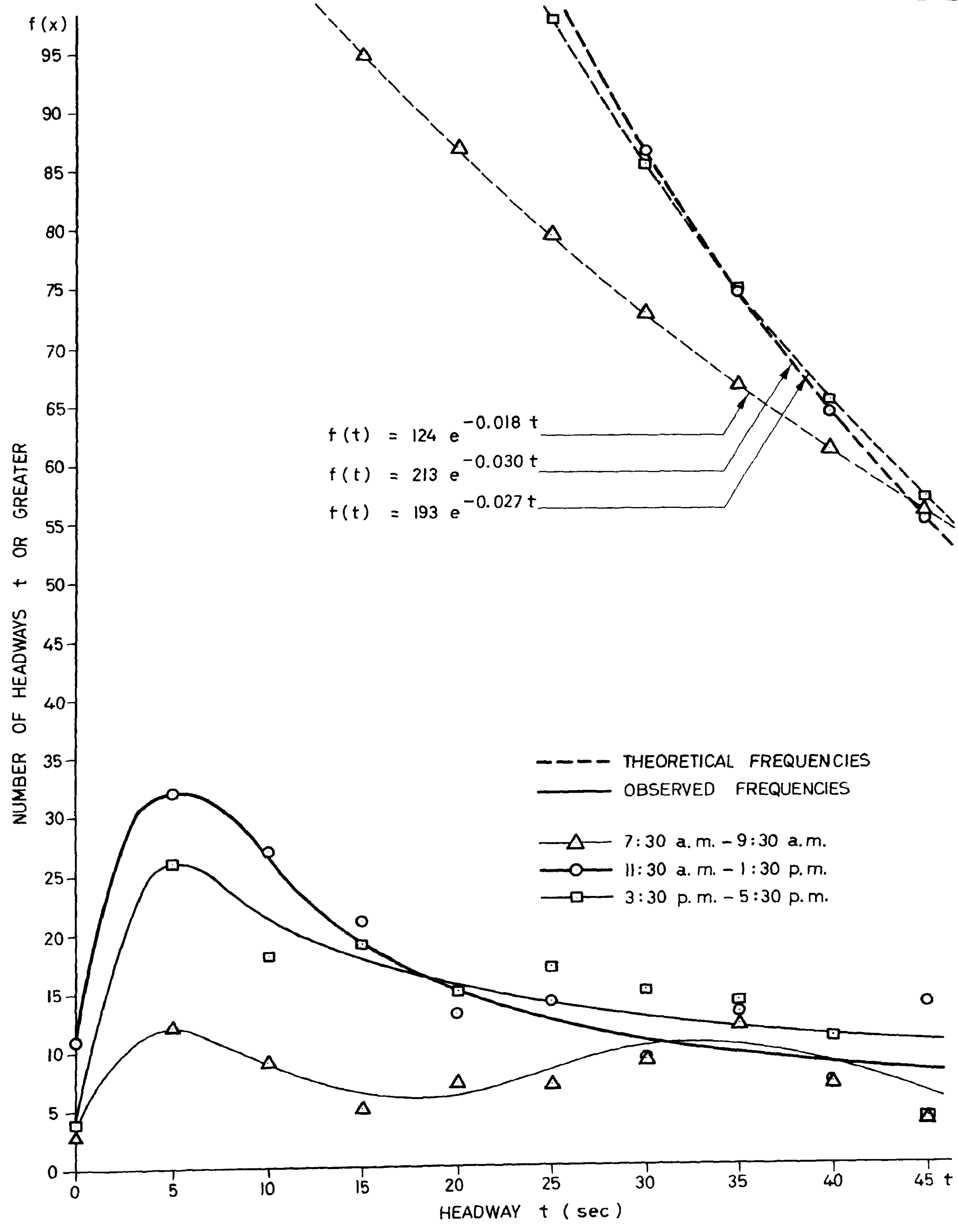

FIGURE 8. NEGATIVE EXPONENTIAL TIME HEADWAY DISTRIBUTION

WEST APPROACH

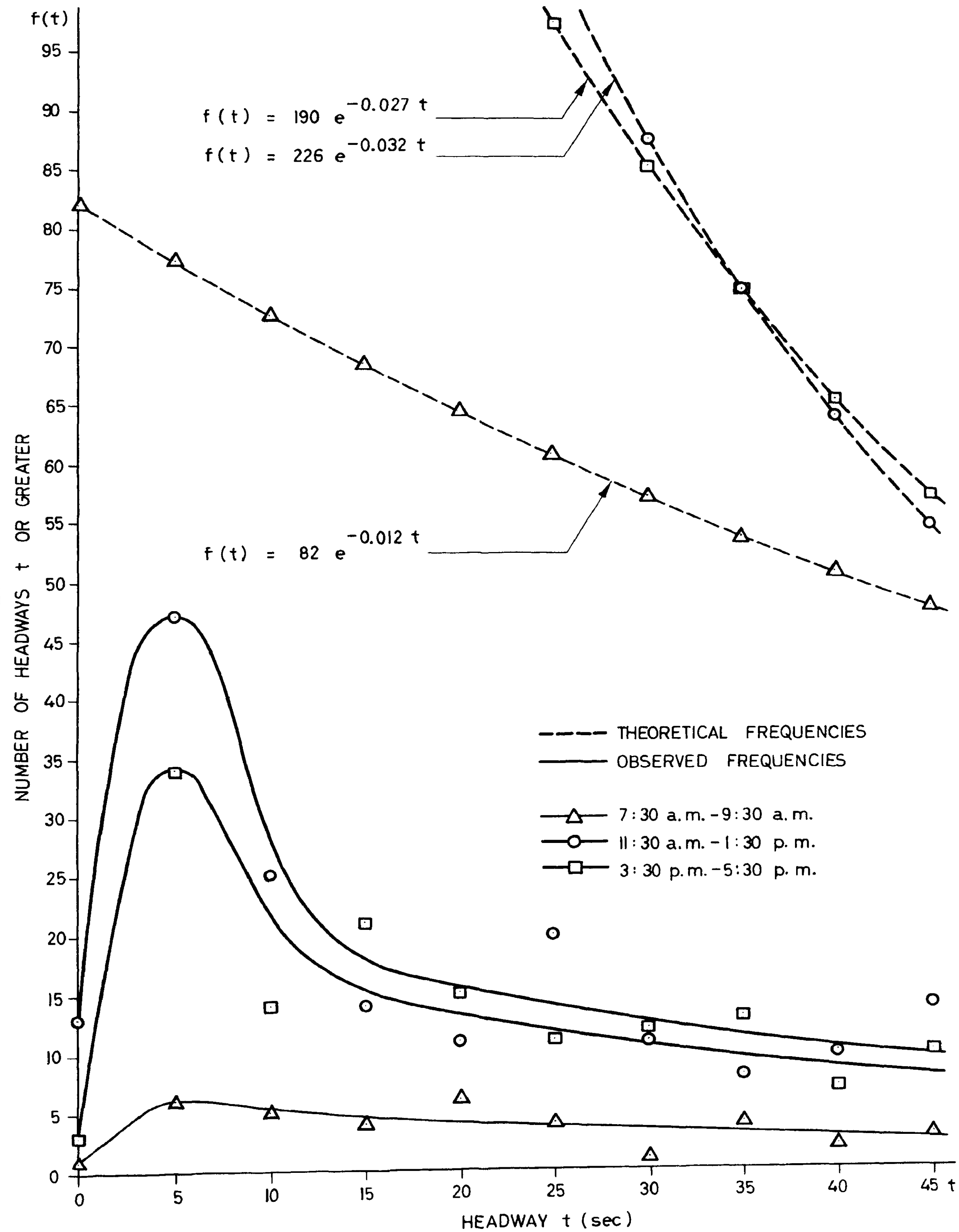

FIGURE 9. NEGATIVE EXPONENTIAL TIME HEADWAY DISTRIBUTION
<u>EAST APPROACH</u>

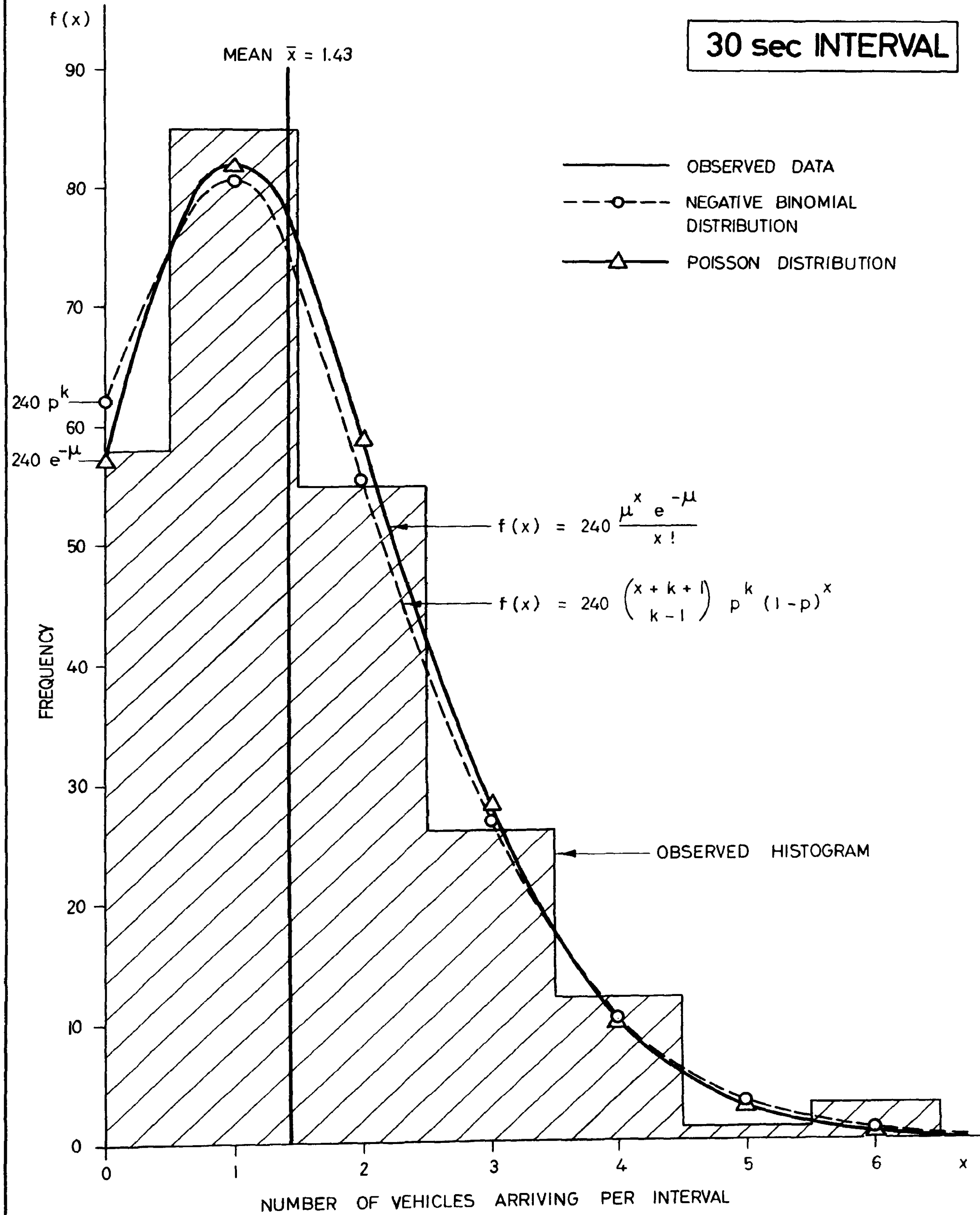

FIGURE 10. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA NORTH APPROACH ; 7:30 a.m.-9:30 a.m.

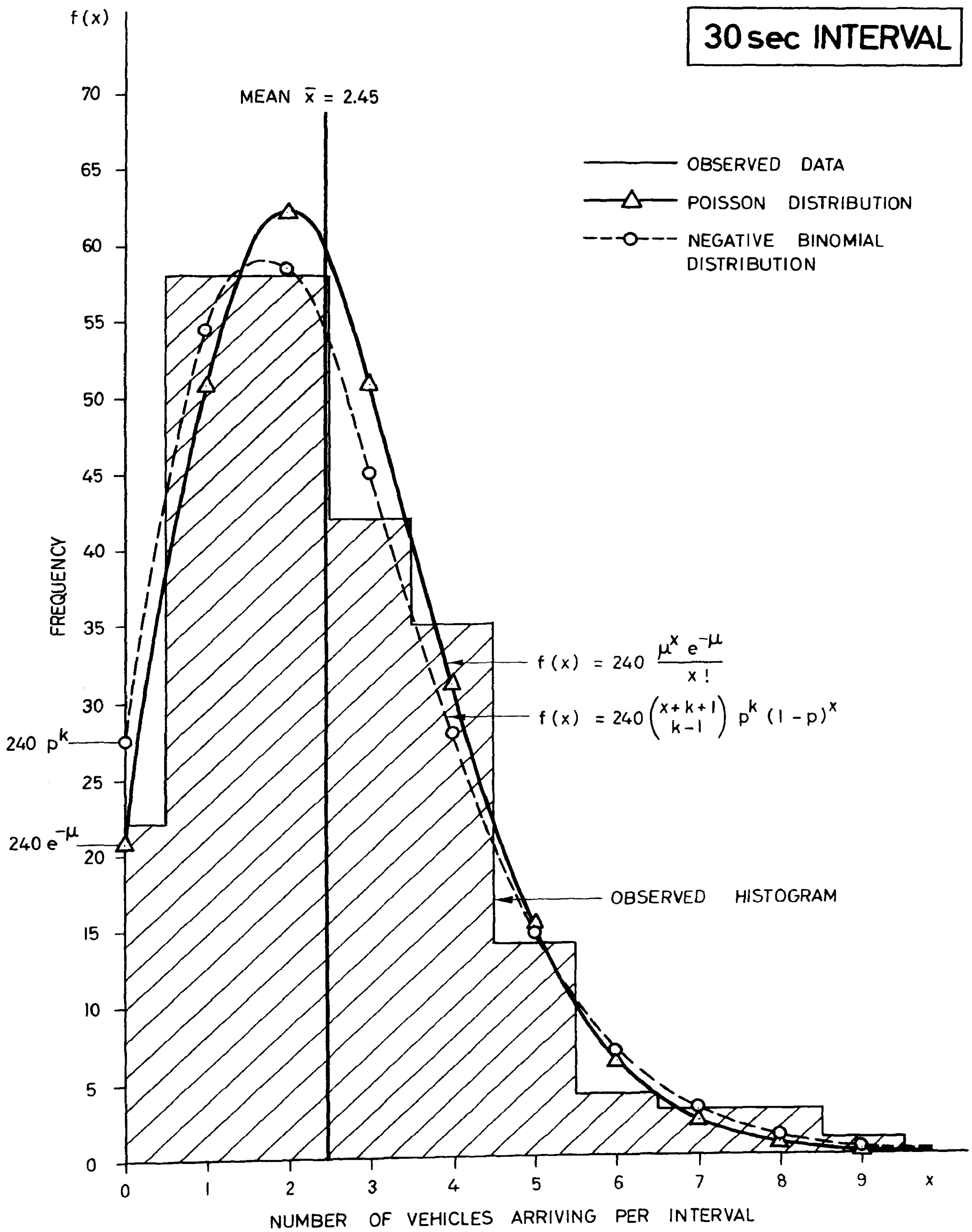

FIGURE 11. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA NORTH APPROACH ; 11:30 a.m.–1:30 p.m.

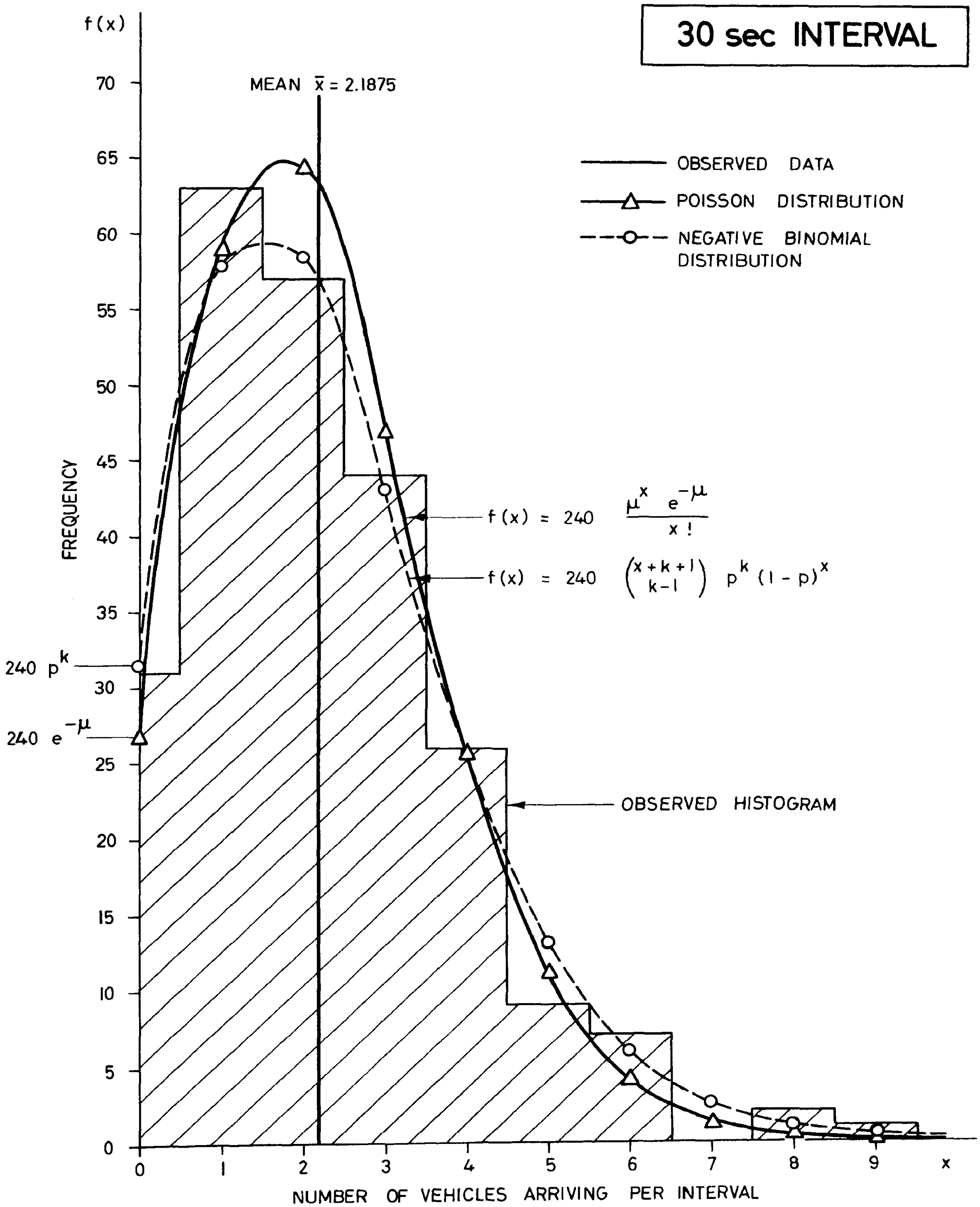

FIGURE 12. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA NORTH APPROACH ; 3:30 p.m. - 5:30 p.m.

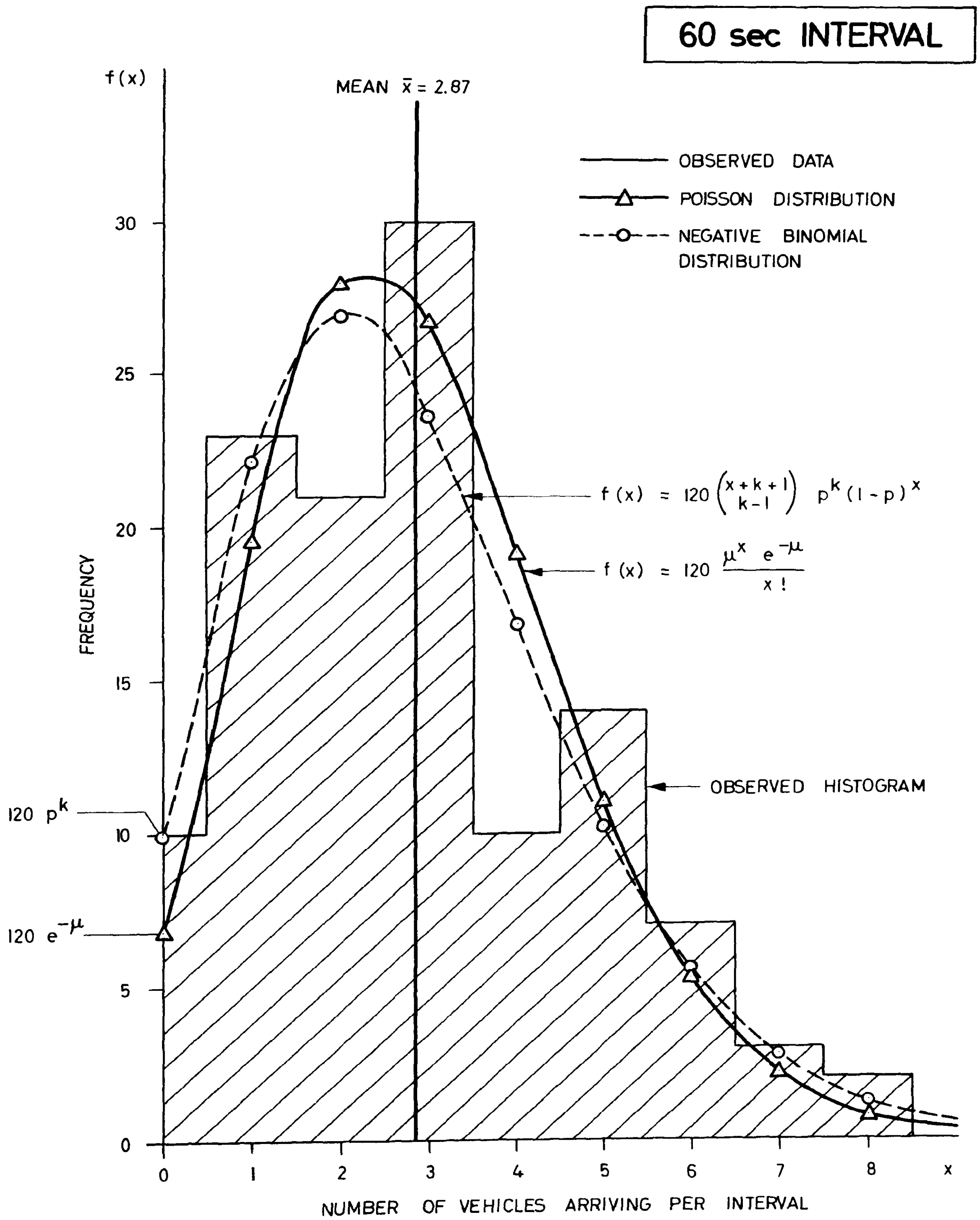

FIGURE 13. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA NORTH APPROACH ; 7:30 a.m.– 9:30 a.m.

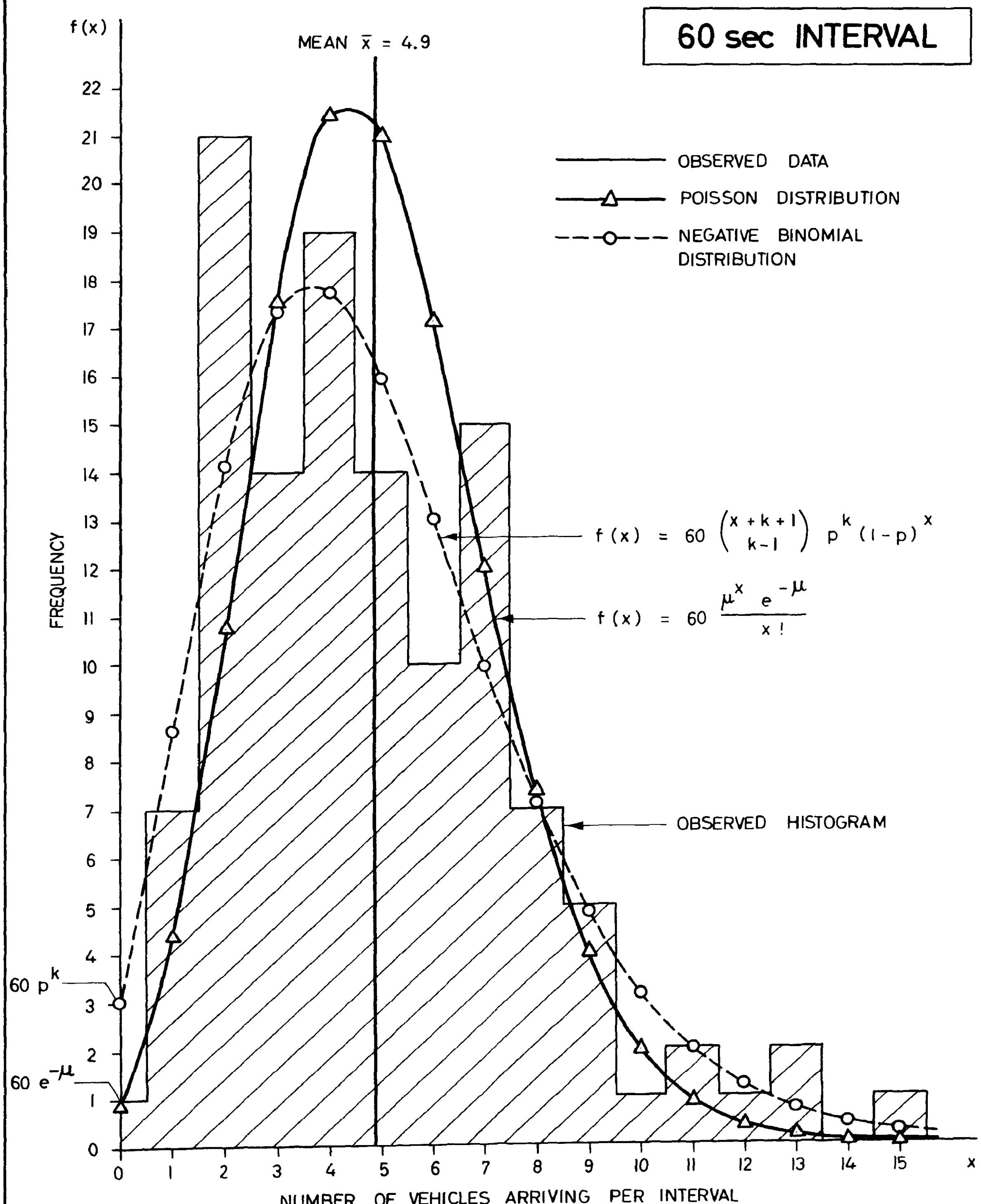

FIGURE 14. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA

NORTH APPROACH ; 11:30 a.m. - 1:30 p.m.

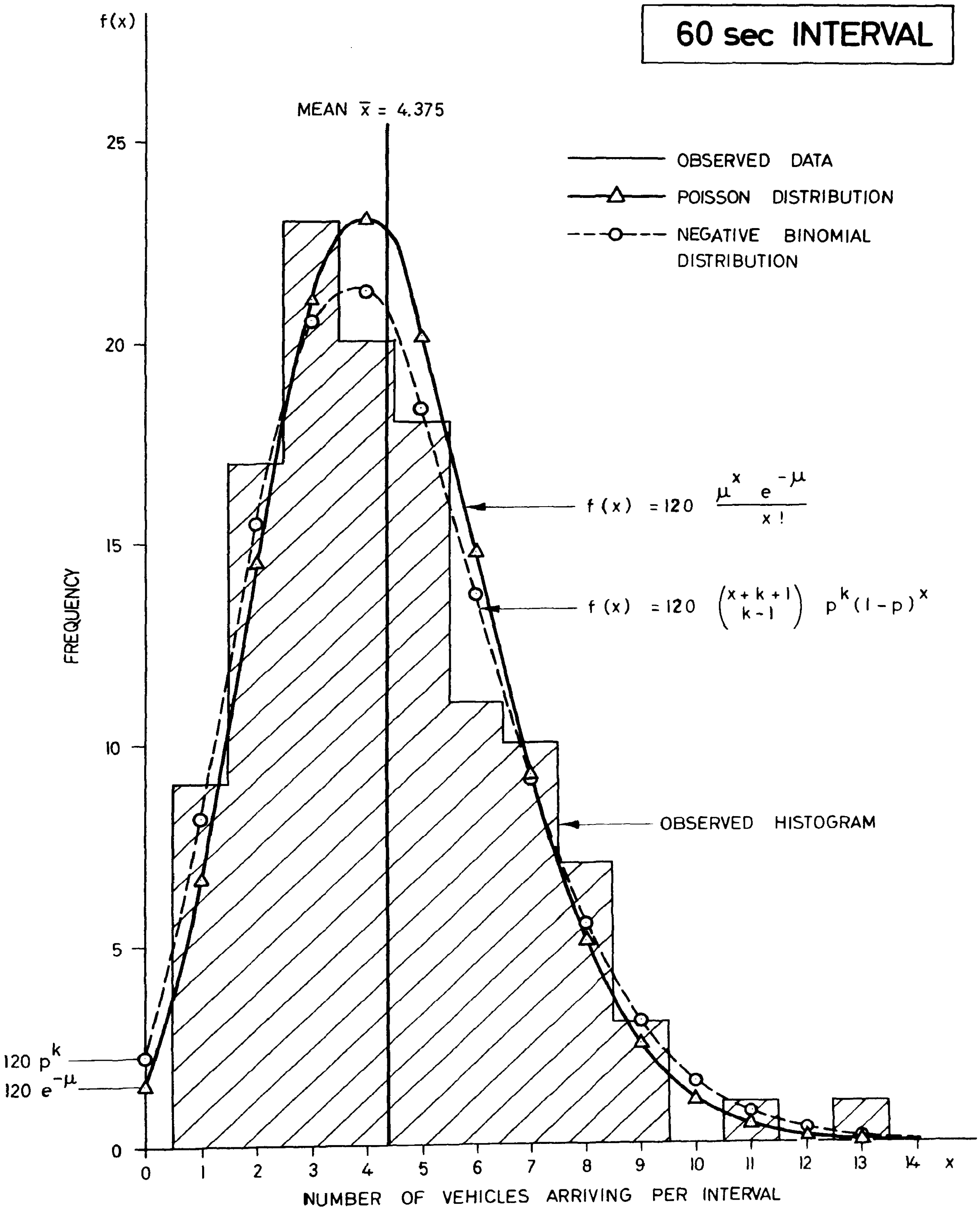

FIGURE 15. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA
NORTH APPROACH ; 3:30 p.m.-5:30 p.m.

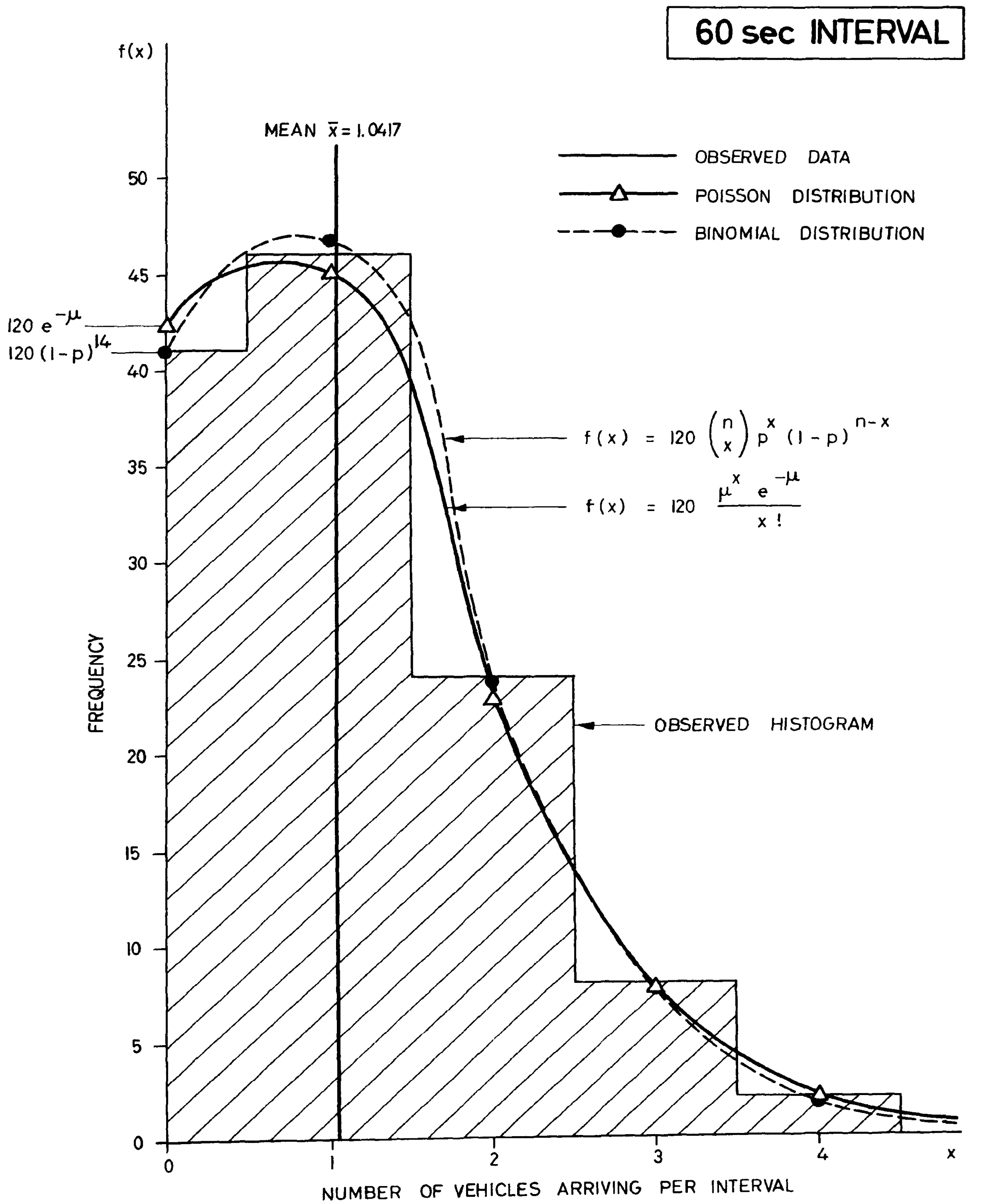

FIGURE 16. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA WEST APPROACH ; 7:30 a.m. - 9:30 a.m.

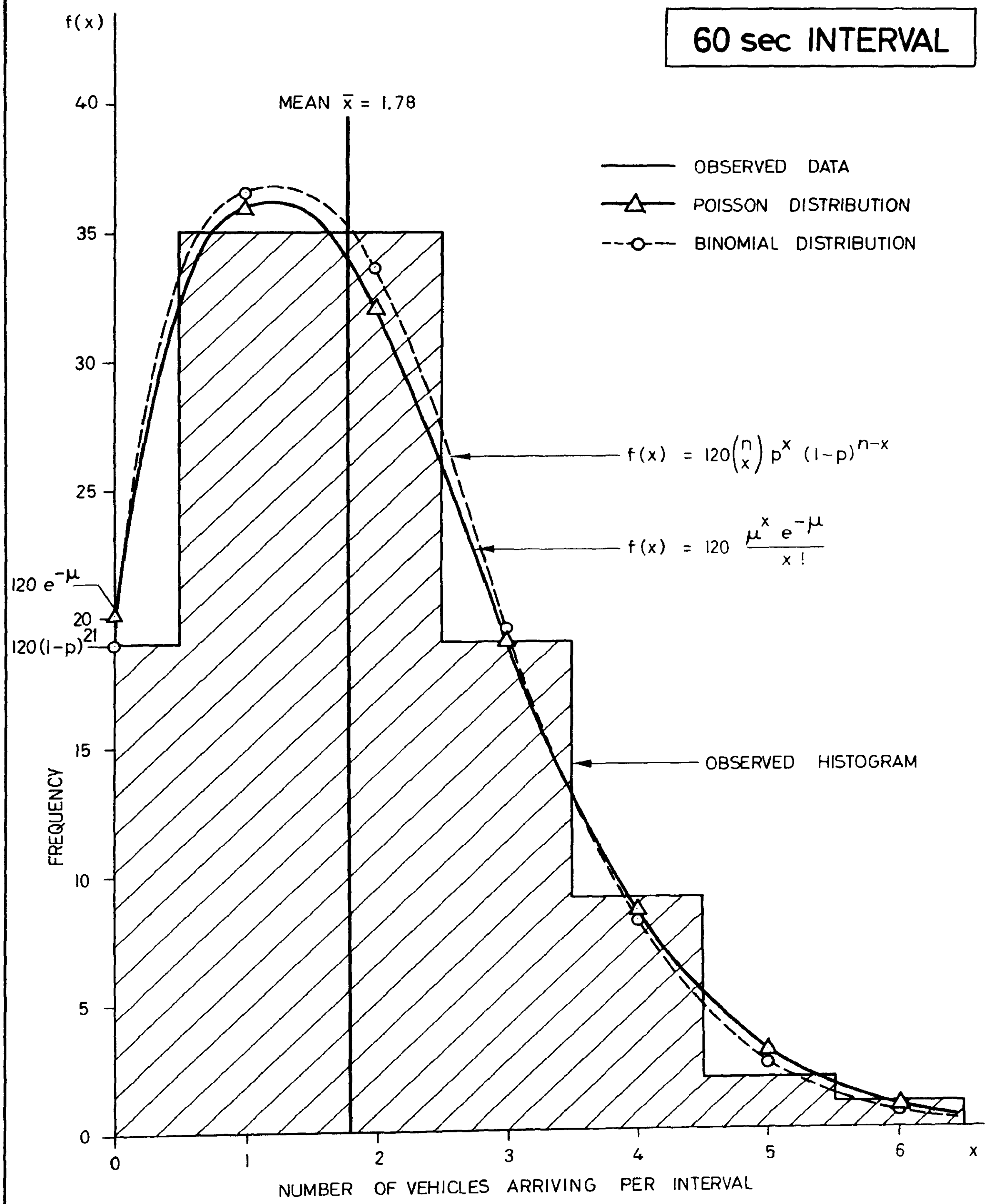

FIGURE 17. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA WEST APPROACH ; 11:30 a.m.–1:30 p.m.

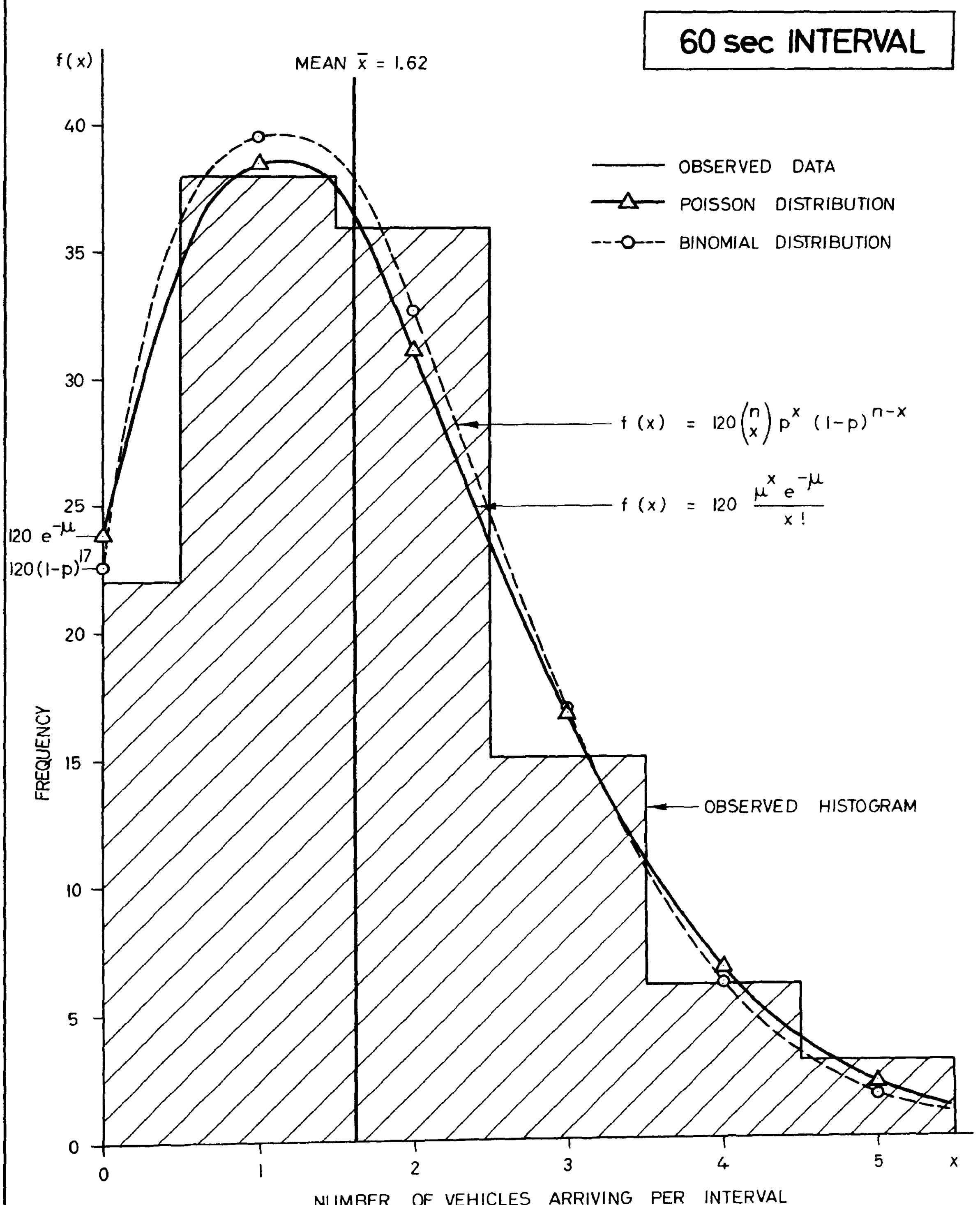

FIGURE 18. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA WEST APPROACH; 3:30 p.m.-5:30 p.m.

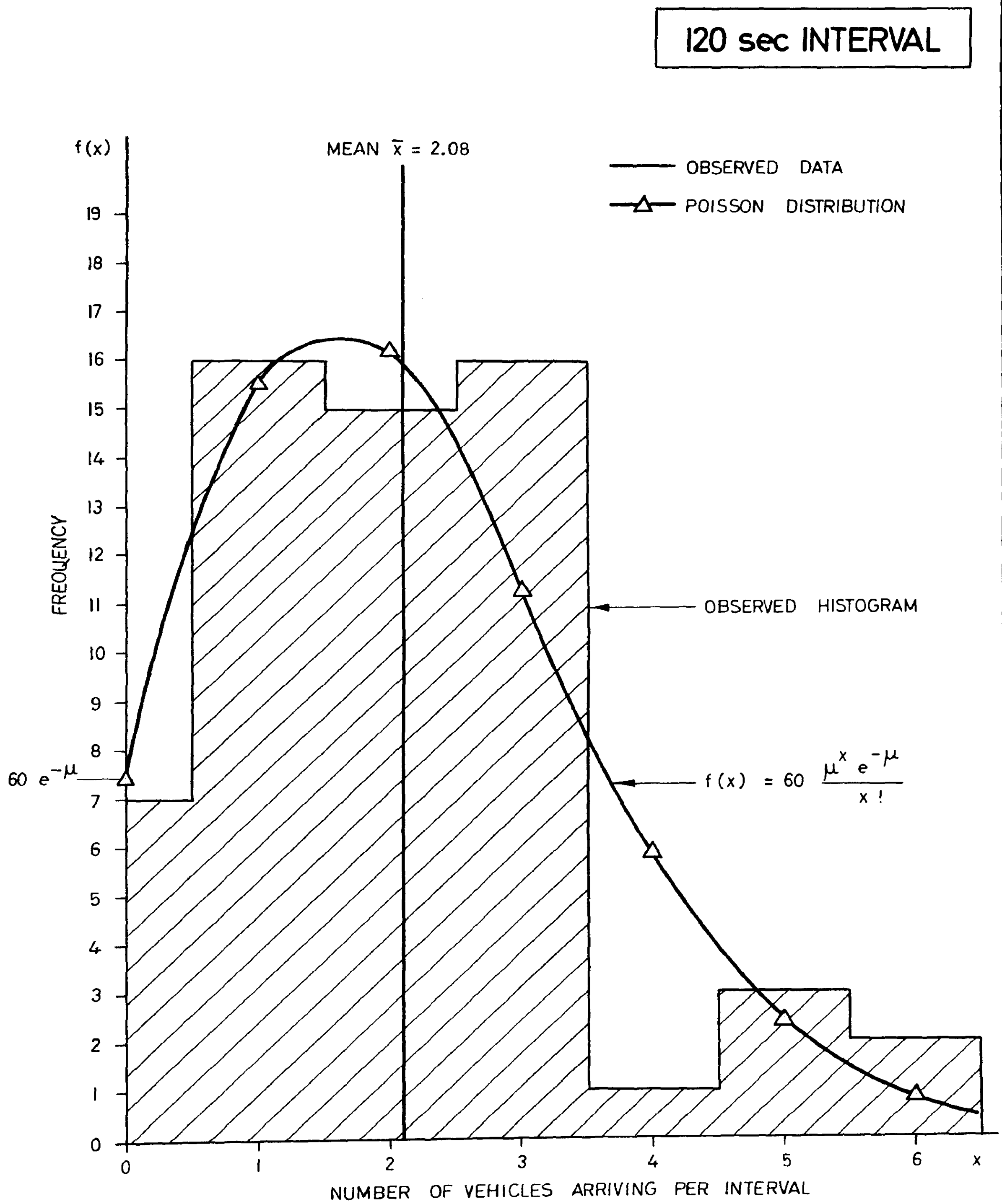

FIGURE 19. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA WEST APPROACH ; 7:30 a.m.–9:30 a.m.

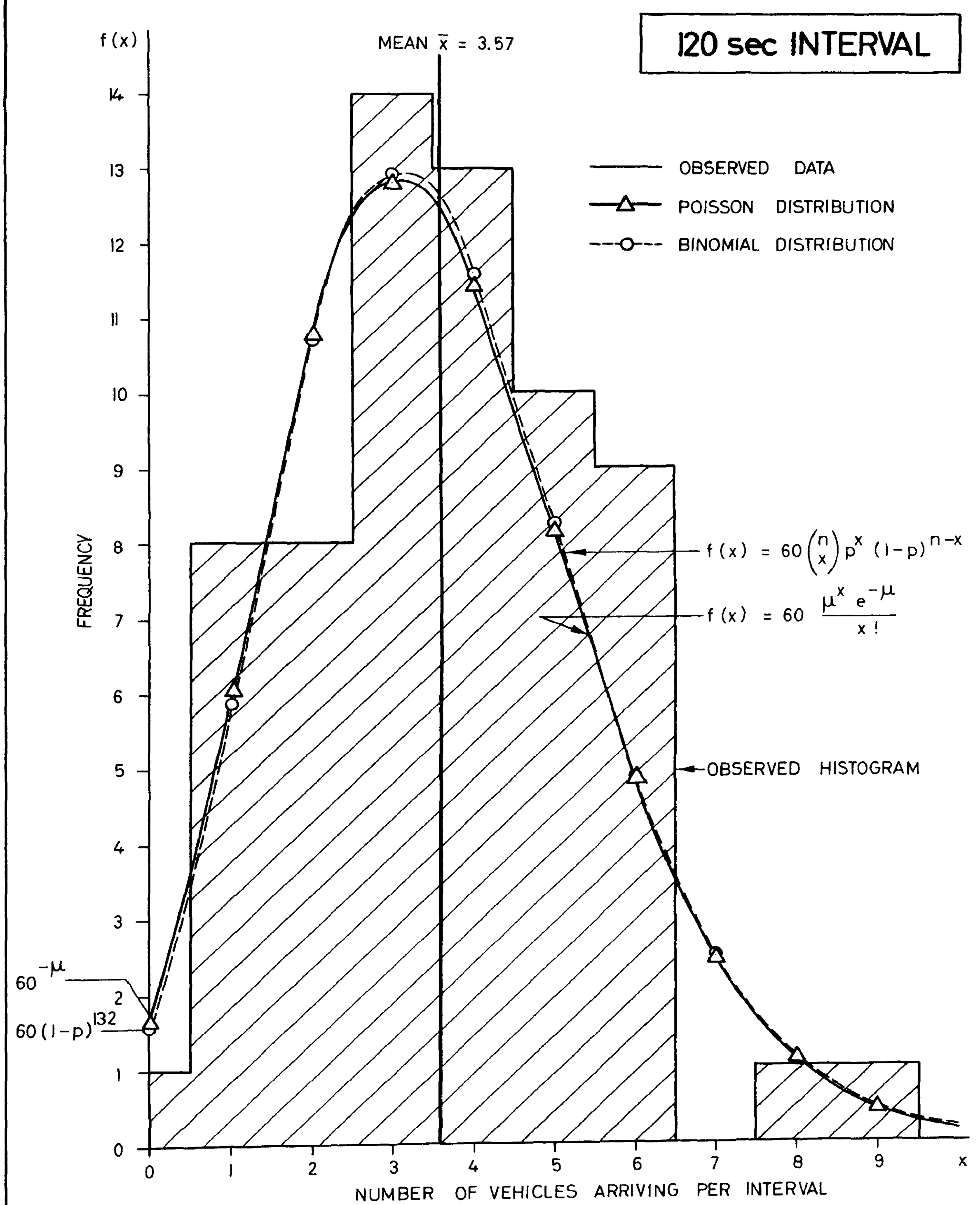

FIGURE 20. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA WEST APPROACH; 11:30 a.m.-1:30 p.m.

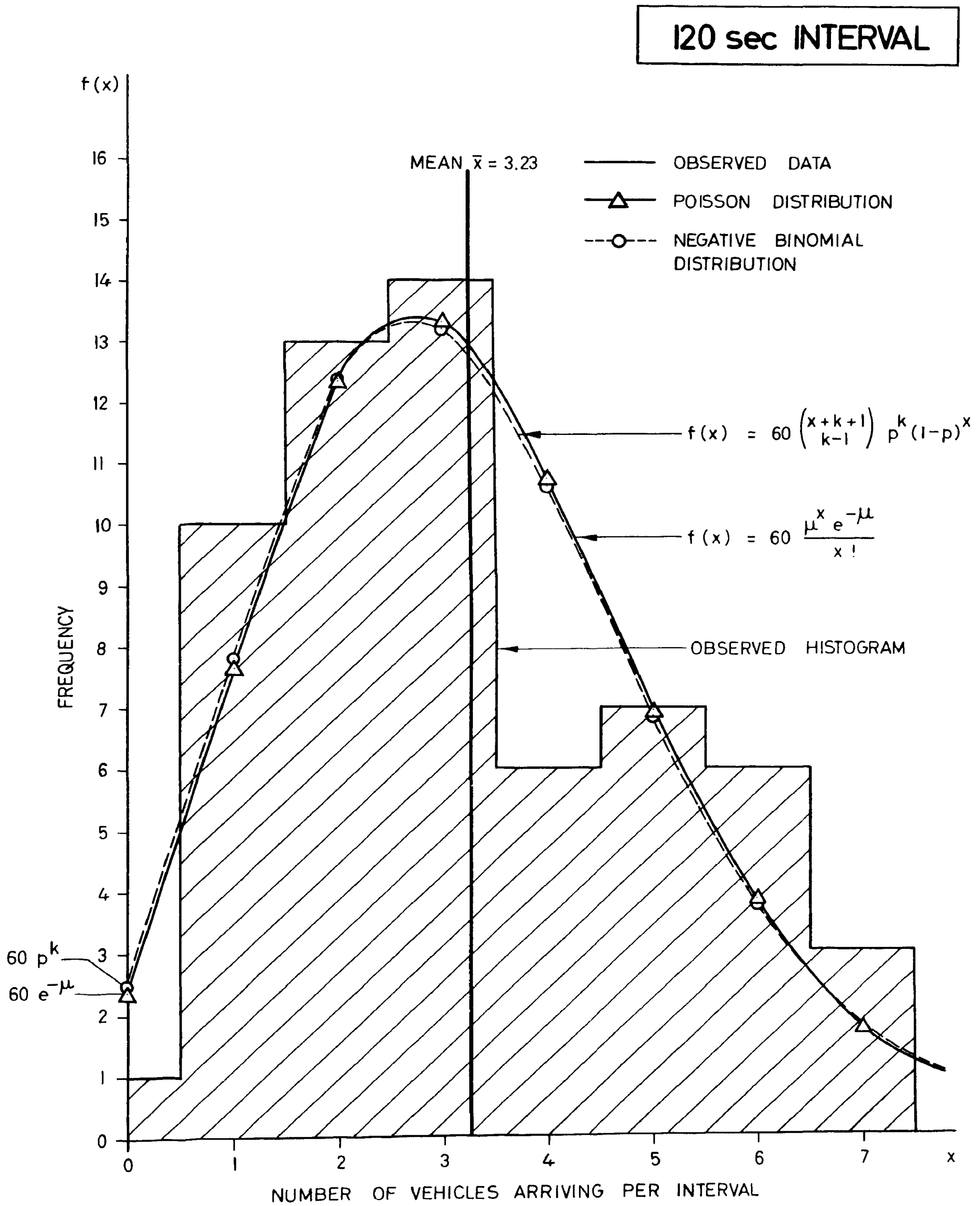

FIGURE 21. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA WEST APPROACH ; 3:30 p.m.–5:30 p.m.

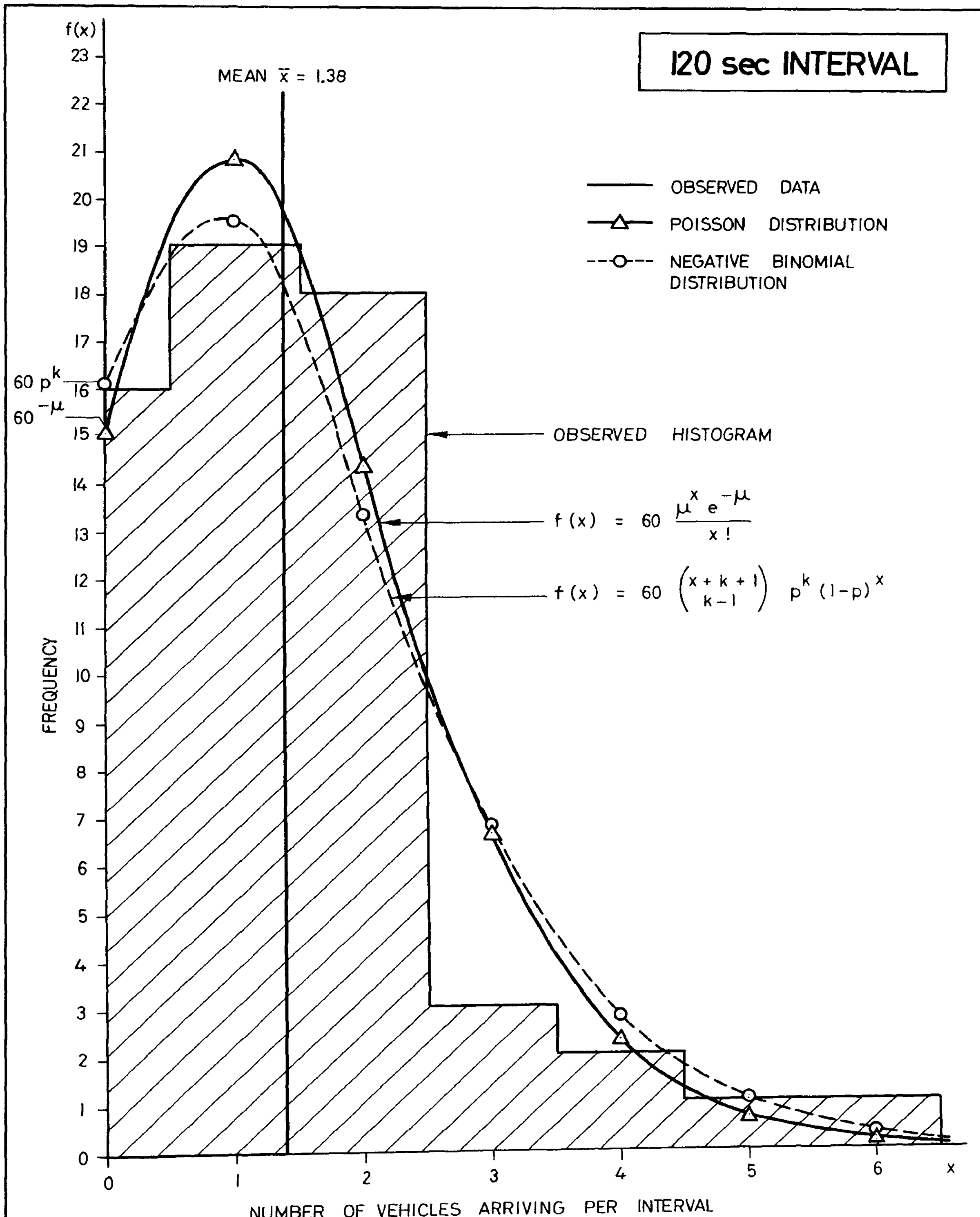

FIGURE 22. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA EAST APPROACH ; 7:30 a.m.–9:30 a.m.

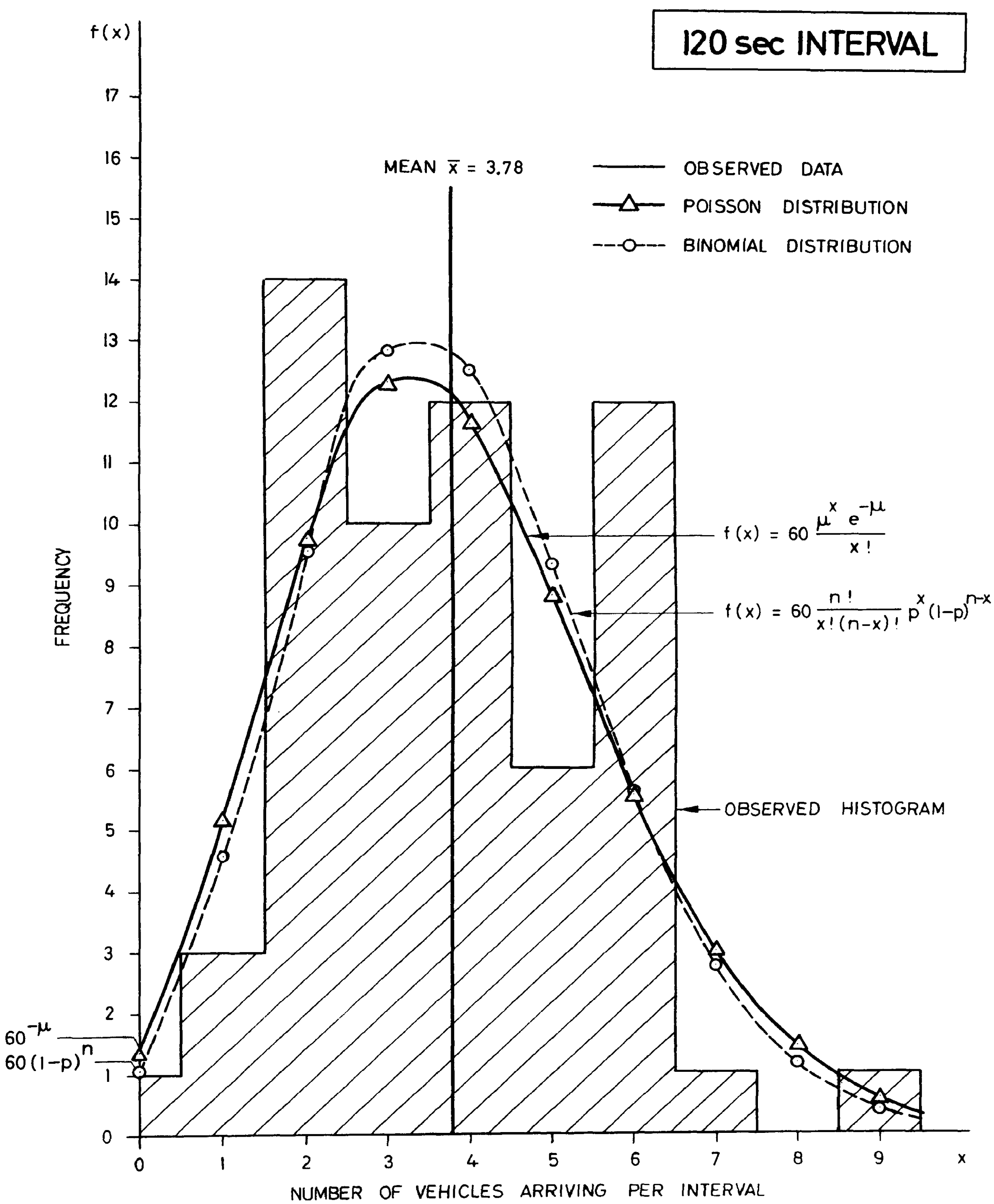

FIGURE 23. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA EAST APPROACH ; 11:30 a.m. – 1:30 p.m.

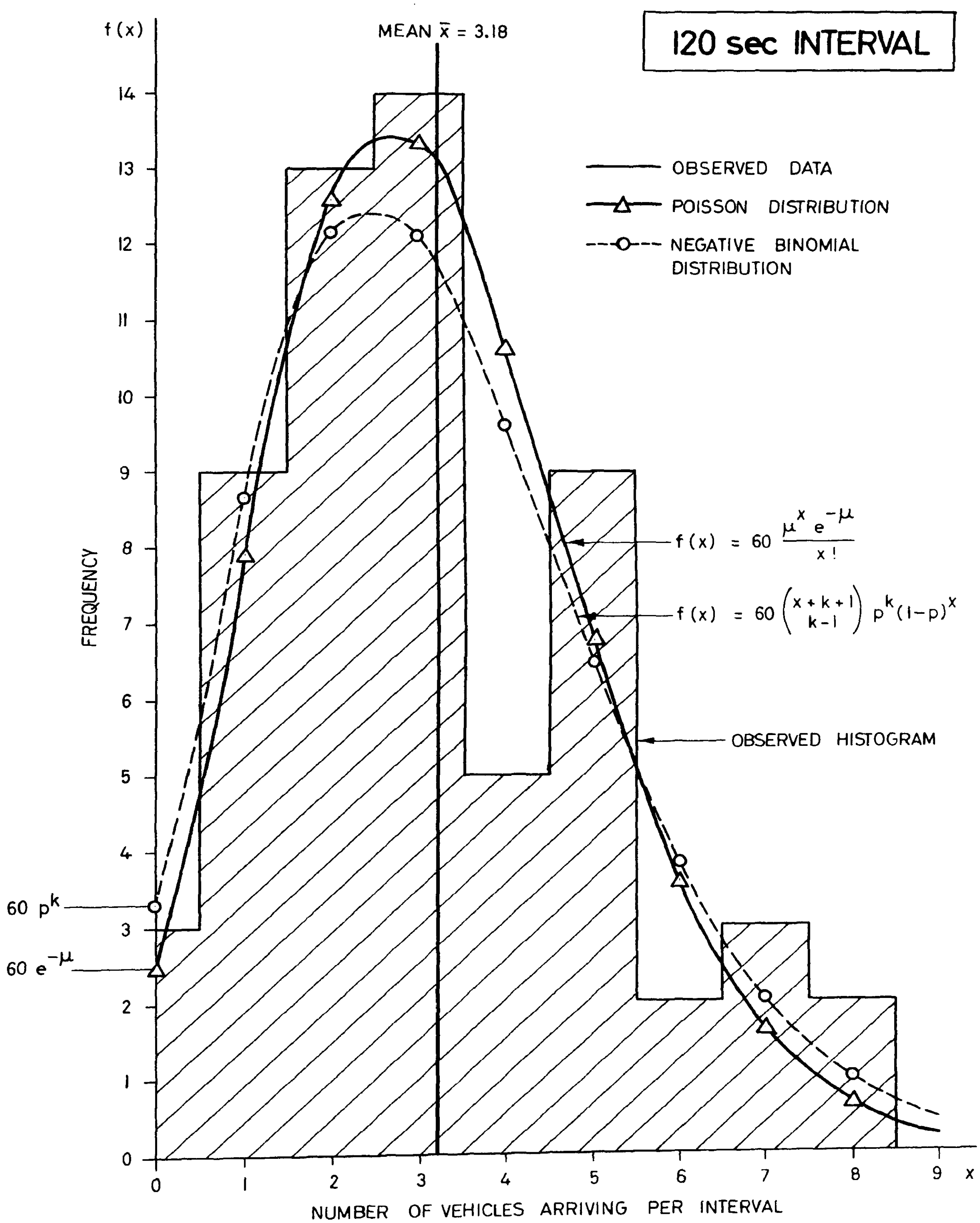

FIGURE 24. FREQUENCY DISTRIBUTION OF VEHICLES ARRIVING AT INTERSECTION ROLLA AND 11th STREET IN ROLLA EAST APPROACH ; 3:30 p.m.–5:30 p.m.

APPENDIX B

(Table XI)

Sample Data Sheet on

Vehicle Arrivals at Intersection Rolla and 11th Street

TABLE XI

VEHICLE ARRIVALS AT INTERSECTION ROLLA AND 11th STREET

FRIDAY, JULY 7, 1972; 7:30 a.m.-9:30 a.m.

ARRIVALS EAST APPROACH

No.	Time	No.	Time	No.	Time
1	7:31:05	29	8:08:10	57	8:57:30
2	31:15	30	09:30	58	59:05
3	32:05	31	12:20	59	9:00:15
4	41:10	32	13:30	60	01:45
5	41:30	33	19:15	61	03:10
6	43:35	34	21:05	62	04:00
7	44:10	35	21:40	63	04:35
8	44:15	36	23:25	64	05:00
9	44:30	37	25:00	65	05:10
10	45:50	38	29:30	66	05:15
11	45:55	39	30:45	67	05:35
12	46:20	40	30:50	68	05:55
13	50:00	41	30:50	69	08:35
14	50:20	42	36:40	70	10:05
15	52:10	43	37:20	71	12:10
16	53:20	44	37:30	72	13:35
17	54:05	45	38:00	73	16:15
18	55:30	46	38:50	74	16:25
19	57:10	47	39:05	75	18:35
20	59:15	48	39:20	76	19:00
21	59:30	49	40:55	77	20:45
22	8:00:25	50	41:00	78	21:05
23	01:10	51	42:05	79	21:25
24	01:20	52	43:30	80	22:00
25	02:55	53	46:50	81	22:05
26	03:20	54	51:10	82	24:30
27	05:40	55	54:35	83	25:15
28	06:20	56	55:55		